Correction to "A Topologically Valid Definition of Depth for Functional Data" by Alicia Nieto-Reyes and Heather Battey

We are grateful to Irène Gijbels and Stanislav Nagy for drawing our attention to some regrettable substantive errors in our paper, which appears in *Statistical Science* **31** 61–79, 2016. With apologies, we present the correct forms below.

1. In Definition 3.1 and in the definitions of band depth and modified band depth on page 68 (lines 15-16, 27-28 and 34) α should be replaced by $\alpha(v)$.
2. The last two lines of Definition 3.1 should be replaced by $U : \mathcal{V} \to \mathfrak{F}$ with $U(v) := \sup_{x \in \mathcal{E}} x(v)$ and $L : \mathcal{V} \to \mathfrak{F}$ with $L(v) := \inf_{x \in \mathcal{E}} x(v)$ when $\max(|U(v)|, |L(v)|) < \infty$ for all $v \in \mathcal{V}$.
3. In Definition 3.2:
   - under P-3., after "exists" should appear "with $D(z,P) = D(z',P)$ implying $d(z,z') = 0$".
   - Equation (3.1) has to be substituted by $\sup_{y \in \mathfrak{F}_x : d(x,y) < \delta} D(y,P) \leq D(x,P) + \epsilon$, where $\mathfrak{F}_x := \left\{ y \in \mathfrak{F} : d(y,x) < d(y,\theta) \text{ or } \max\{d(y,\theta), d(y,x)\} < d(x,\theta) \right\}$ for $\theta = \operatorname{argsup}_{x \in \mathfrak{F}} D(x,P)$.
   - In P-5., $\mathfrak{C}(\mathfrak{F},P)$ is substituted by $\mathfrak{C}(\mathfrak{F},P)\backslash 0$ and the interval of definition of δ by $[\inf_{v \in \mathcal{V}} d(L(v), U(v))), d(L,U)) \cap (0,\infty)$.
4. Lemma 4.3 is false. Consequently, there is a cross in the corresponding position in Table 2. *Conter-example:* Let $(\mathfrak{F}, d) = (\mathbb{H}, \|\cdot\|_{\mathbb{L}_2})$ and P a discrete distribution on $\mathfrak{F}$ with support $\{X_1, X_2, X_3\}$ such that $P(X_1) = P(X_3) = 1/4$ and $P(X_2) = 1/2$ and $d(X_1,X_2) = d(X_2,X_3) = d(X_1,X_3)/2$. Let $x \in \mathfrak{F}$ such that $d(x, X-1) = d(x, X_2) = d(X_1,X_2)/2$, then, there always exist a h such that $D_h(x,P) < \min(D_h(X_1,P), D_h(X_2,P))$. This is due to $D_h(x,P) = \mathbb{E}[\exp(-(x/h)^2/2)/(h\sqrt{2\pi})]$.

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