Corrigendum: Asymptotic spectral theory for nonlinear time series

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In this note, we make a correction to an error in Lemma A.4 of Shao and Wu (2007), as pointed out by Professor Stathis Paparoditis.

In LEMMA A.4 (i), the original statement is
\[
\max_{j,k \leq m} |\text{cov}(I_j^2, I_k^2) - 4f_j^4\delta_{j,k}| = O(1/n).
\]

There is an error in the constant 4, and the correct statement should be
\[
\max_{j,k \leq m} |\text{cov}(I_j^2, I_k^2) - 20f_j^4\delta_{j,k}| = O(1/n).
\]

As in the original proof, we only need to consider indecomposable partitions [Rosenblatt (1985, page 34)] consisting of 4 sets, each containing 2 Xs (one with positive sign and the other with negative sign). In total there are 20 such partitions. In the original proof, 4 of them are considered, which are
\[
\{(t_1, s_1), (t_2, s_2), (t_3, s_3), (t_4, s_4)\}, \{(t_1, s_1), (t_2, s_2), (t_3, s_3), (t_4, s_4)\},
\]
\[
\{(t_1, s_1), (t_2, s_2), (t_3, s_3), (t_4, s_4)\}, \{(t_1, s_1), (t_2, s_2), (t_3, s_3), (t_4, s_4)\}.
\]

Each partition leads to the sum \([A(\lambda_j, \lambda_k)]^4 = f_j^4\delta_{j,k} + O(1/n)\), where \(A(\lambda_j, \lambda_k) = \frac{1}{2\pi n} \sum_{t_1, s_1=1}^n r(t_1 - s_1)e^{ijt_1^\lambda}e^{ikt_1}, \) and \(r(k) = \text{cov}(X_1, X_{k+1})\).

As suggested by Professor Paparoditis, there are 16 more indecomposable partitions, 4 of them are
\[
\{(t_1, t_2), (s_1, s_2), (t_3, s_3), (t_4, s_4)\}, \{(t_1, t_2), (s_1, s_2), (t_3, s_3), (t_4, s_4)\},
\]
\[
\{(t_1, t_2), (s_2, s_3), (t_3, s_1), (t_4, s_4)\}, \{(t_1, t_2), (s_2, s_3), (t_3, s_1), (t_4, s_4)\}.
\]

Each partition leads to the sum \(A(\lambda_j, \lambda_j)A(\lambda_k, \lambda_k)\) \([A(\lambda_j, \lambda_k)]^2 = f_j^4\delta_{j,k} + O(1/n)\).

The other 12 partitions correspond to
\[
\{(t_1, t_3), (s_1, s_2), (t_3, s_3), (t_2, s_4)\} \quad \{(t_2, t_3), (s_1, s_2), (t_1, s_3), (t_4, s_4)\}
\]
\[
\{(t_3, t_4), (s_1, s_2), (t_1, s_3), (t_2, s_4)\} \quad \{(t_1, t_4), (s_1, s_2), (t_3, s_3), (t_2, s_4)\}
\]

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and they all lead to $f_j^4\delta_{j,k} + O(1/n)$.

Another way to heuristically check that the constant factor in

$$\max_{j,k \leq m} |\text{cov}(I_j^2, I_k^2) - c f_j^4 \delta_{j,k}| = O(1/n)$$

is indeed $c = 20$ is to observe that when $j = k$, $\text{var}(I_j^2)/f_j^4 = \text{var}([I_j/f_j]^4]$ and $I_j/f_j$ follows EXP(1) distribution asymptotically. Then we have $\text{var}(I_j/f_j)^2 = E(I_j/f_j)^4 - [E(I_j/f_j)^2]^2 \approx 4! - (2!)^2 = 20$, so the factor $c$ is expected to be 20.

Many thanks to Professor Paparoditis for pointing out this 15-yr old error!

References
