Discussion contribution to paper

N. Nolde & J. F. Ziegel: Elicitability and Backtesting: Perspectives for Banking Regulation.

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The approach to back-testing proposed in this paper is to replace the traditional significance test by a test of the performance of a given model against a 'standard model' as an alternative. From a theoretical point of view this meshes well with analysis based on elicitability/identifiability, particularly now that the objections to Expected Shortfall have been disposed of. From the regulatory perspective it surely makes sense to reject a model if it cannot outperform a simple alternative, and the authors' work could be interpreted as a formalization of conventional regulatory thinking. The connection is helped by the 3-colour traffic light matrices, which provide a very intuitive way of interpreting the test results.

As the authors point out, the work is related to my own paper (Davis, 2016) on verification of risk measure estimates. The idea there was to recognize that the conditional calibration condition given in Definition 3 of the present paper is saying that $\{Y_t \equiv V(R_t, X_t), t = 1, 2, \ldots\}$ is a martingale difference sequence and to state conditions under which the martingale SLLN can be applied to imply convergence of various normalized sums when the predictor is correctly calibrated. Value-at-Risk plays a wholly special role here, in that the Probability Integral Transform (PIT) can be applied to replace the sequence of exceedances by X_t of the β-quantile threshold by exceedances of β by an i.i.d. sequence of uniform $[0, 1]$ random variables. Then the ordinary SLLN and/or LIL can be applied, essentially no conditions being required. In other cases the martingale SLLN and CLT depend on properties of the conditional quadratic variation process $\langle Y \rangle_t$, but this is not directly observed, leading to a somewhat artificial definition of the class $\mathcal{P}$ specifying the universe of possible models under discussion. Nolde and Ziegel follow Giacomini and White (2006) in defining test statistics such as T_1 of (2.11) which brings in a sample covariance estimate $\hat{\Omega}_n$ (as opposed to the conditional quadratic variation). Under some conditions T_1 has an asymptotic χ^2 distribution and a rejection region can be defined. Nolde and Ziegel do not discuss these conditions, but they are a mixing condition (going back to extensive earlier work by authors including MacLeish[1]) together with moment and positivity conditions. They are not unreasonably restrictive and do allow the statistical analysis to proceed, but in the context of financial data there could be some debate about the degree of stationarity they imply, see the remarks in MacLeish's paper. Also, they are as artificial as those complained about above, but that seems unavoidable in problems of this type.

In summary, the authors are to be congratulated on an excellent paper, which in my view makes an important conceptual contribution to the development of back-testing procedures.

[1] D.L. MacLeish, On the invariance principle for nonstationary mixingales, *Ann. Prob.* 5 (1977) 616-621