

SIMULTANEOUS MODELLING OF MOVEMENT, MEASUREMENT ERROR, AND OBSERVER DEPENDENCE IN MARK-RECAPTURE DISTANCE SAMPLING: AN APPLICATION TO ARCTIC BIRD SURVEYS

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Mark-recapture distance sampling is a promising method for surveying bird populations from aircraft in open landscapes. However, commonly available distance sampling estimators require that distances to target animals are made without error and that animals are stationary while sampling is being conducted. Motivated by a recent bird survey where these requirements were routinely violated, we describe a marginal likelihood framework for estimating abundance from double-observer data that can accommodate movement and measurement error when observations are made consecutively (as with front and rear observers), when animals are uniformly distributed during detection by the first observer, and when detections consist of both moving and stationary animals. Assuming that all animals are subject to measurement error and that some animals can move between detections, we integrate over unknown animal locations to construct a marginal likelihood for detection, movement, and measurement error parameters. Estimates of animal abundance are then obtained using a modified Horvitz-Thompson-like estimator. In addition, unmodelled heterogeneity in detection probability can be accommodated through observer dependence parameters. Using simulation, we show that our approach yields low bias compared to approaches that ignore movement and/or measurement error, including in cases where there is considerable detection heterogeneity. Applying our approach to data from a double-observer waterfowl helicopter survey in northern Canada, we are able to estimate bird density accounting for movement and measurement error and corrected for observer heterogeneity. Our approach appears promising for generating unbiased estimates of bird abundance necessary for reliable conservation and management.

- 1 **1. Introduction.** Distance sampling surveys ([Burnham, Anderson and](#)
2 [Laake, 1980](#); [Buckland et al., 2001](#)) are often used to estimate the abundance

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3 of wildlife populations. Such surveys were historically conducted by a sin-
4 gle observer who followed a transect line and recorded the perpendicular
5 distance to each detected animal group. Assuming 100% detection on the
6 transect line, investigators can fit models to estimate abundance over the
7 surveyed area while accounting for detection probabilities that decline with
8 distance from the transect line.

9 When surveys are conducted from the air, double-observer surveys have
10 major advantages over single-observer surveys. For instance, one can use
11 records of detection/non-detection to relax the assumption of perfect detec-
12 tion on the transect line (Borchers, Zucchini and Fewster, 1998). Analysis
13 of double-observer distance data is now canonically referred to as “mark-
14 recapture distance sampling” (MRDS; Laake and Borchers, 2004) because
15 there is a detection history (i.e. binary detection/nondetection records for
16 each observer) in addition to recorded distances.

17 Despite the potential usefulness of MRDS methods for estimating abun-
18 dance, published methods often assume that animals do not move between
19 detections of observers, and that distances are recorded without error. These
20 assumptions are problematic for some species and sampling situations, such
21 as aerial surveys of birds where individuals may exhibit responsive move-
22 ment away from aircraft. Several authors have investigated consequences and
23 corrections for movement in distance sampling applications. For instance,
24 Glennie, Buckland and Thomas (2015) showed that movement could cause
25 considerable bias (typically positive) in distance-based abundance estima-
26 tors, but did not attempt to develop methods to adjust for such bias. Hiby
27 and Lovell (1998) developed a likelihood framework to estimate abundance
28 when movement is random (i.e. nonresponsive to the survey platform) and
29 occurs between successive observations; however, their approach does not
30 account for responsive movement away from the survey platform.

31 Likewise, Borchers et al. (2010) showed that measurement errors could
32 cause substantial (usually positive) bias in distance sampling abundance es-
33 timators. A number of authors have proposed models that account for mea-
34 surement error in specific distance sampling applications (see e.g. Schweder
35 et al., 1999; Borchers et al., 2010, and references therein).

36 Several observer configurations are possible within an MRDS estimation
37 framework (Burt et al., 2014) and have important implications for bias con-
38 trol when animals move in response to a survey platform (i.e. “responsive”
39 movement). In an “independent” configuration, observers detect animals in-
40 dependently of one another. Under this configuration it is possible to try
41 to account for heterogeneity in detection probabilities (e.g. visual distinc-
42 tiveness of different animal groups) by modelling lack of fit between the

43 distribution of observed distances and estimated detection probabilities as
44 a function of distance (Laake and Borchers, 2004; Borchers et al., 2006;
45 Buckland, Laake and Borchers, 2010). The ability to account for such het-
46 erogeneity is important because abundance estimators are negatively biased
47 otherwise. Alternatively, in a “trial” configuration (Buckland and Turnock,
48 1992), one observer searches ahead, while another searches closer to the sur-
49 vey platform. Under this configuration, detections by the first observer are
50 used as trials for the second observer. The trial configuration is useful for
51 reducing bias associated with responsive movement of animals (which of-
52 ten positively biases abundance estimators), but one can no longer model
53 heterogeneity in detection probability (Burt et al., 2014).

54 In this paper, we develop an integrated likelihood framework to account
55 for movement and measurement error using an independent observer MRDS
56 configuration. Our development is motivated by aerial surveys of birds,
57 where the goal is to unbiasedly estimate the species densities needed for
58 effective monitoring and conservation decisions. As such, we address move-
59 ment between the time that two observers (e.g. front and rear seat observers
60 on the same side of the aircraft) are able to make detections. Our objective
61 is to account for the biasing effects of measurement error and responsive
62 movement while also being able to model individual heterogeneity through
63 an observer dependence specification. The remainder of this article is struc-
64 tured as follows. First, we describe a motivating data set, in which distances,
65 detection histories, and individual covariates are summarized from a double-
66 observer aerial survey in northern Canada. Second, we describe a maximum
67 marginal likelihood (MML) framework for analyzing these data. Under this
68 framework, true animal locations are treated as latent variables. Next, we
69 illustrate our method by analyzing the waterfowl data set and examine es-
70 timator performance with two simulation studies. We conclude with a short
71 discussion on our proposed modeling framework, future research needs, and
72 some specific suggestions related to bird survey applications.

73 **2. Waterfowl data.** In June of 2014, biologists conducted a pilot double-
74 observer helicopter (BELL 206L on floats) survey of Arctic bird species in
75 the Queen Maud Gulf Migratory Bird Sanctuary (Nunavut, Canada; Fig.
76 1). The total length of survey tracks was 947.8km. Birds surveyed were pre-
77 dominantly waterfowl, but also included cranes and ptarmigan; we refer to
78 them collectively as waterfowl for the remainder of the paper. The original
79 intent of the survey was to investigate the potential of MRDS methods for
80 surveying Arctic waterfowl, and to compare them to other commonly used
81 waterfowl survey methods such as those that use double observers but don’t

82 record distance (e.g. [Koneff et al., 2008](#)). It was thus conducted opportunistically, lacking a sampling frame needed for extrapolation to larger areas.
83
84 The survey is described in greater detail elsewhere ([Alisauskas and Conn, 2017](#)), but we briefly provide information relevant to the analysis conducted
85
86 later in this paper.

87 During the survey, two observers, one behind the other, on the same (left)
88 side of the helicopter independently detected and recorded the perpendicular
89 distance from the transect line to each bird group they observed. Distances
90 were binned into 6 classes: 0-40m, 40-80m, 80-120m, 120-160m, 160-200m,
91 and 200m+ (note that observations in the final bin are not used in subse-
92 quent analysis). They also recorded species, the number of waterfowl in each
93 detected group (“group”), and a binary indicator for whether the waterfowl
94 group was flapping their wings (“moving”). These data were previously an-
95 alyzed by [Alisauskas and Conn \(2017\)](#), who used standard MRDS methods
96 that ignored movement and measurement error in their analysis. Their anal-
97 ysis suggested that moving birds were more detectable than stationary ones,
98 that detection probability increased with group size, and that the front seat
99 observer had higher detection probability than the rear seat observer. They
100 also estimated similar species effects on detection for 7 of the 9 species an-
101 alyzed; here, we analyze detections of these 7 species (Canada goose, king
102 eider, long-tailed duck, northern pintail, rock ptarmigan, sandhill crane, and
103 white-fronted goose), within the same model to illustrate modeling concepts.
104 This protocol led to a total of 964 unique waterfowl group detections; 359
105 were detected by both observers, 348 by the front observer only, and 257 by
106 the back observer only. Note that the back seat observer’s view of the first
107 distance bin nearest the transect line was partially obstructed by the left
108 helicopter float. A plot of observed distance deviations suggested asymmet-
109 rical responsive movement (away) from the aircraft for nonstationary animal
110 groups. There were also some minor distance discrepancies for animal groups
111 that were not moving, suggesting measurement error (Fig. 2).

112 The observed distribution of distances is problematic in the sense that
113 it explicitly contradicts the standard MRDS modeling assumption that an-
114 imals do not move between successive observations. Further, they make in-
115 dividual heterogeneity difficult to diagnose because the patterns in distance
116 data that are indicative of heterogeneity can be obscured by patterns due
117 to distance. This leads to some natural questions: first, what degree of bias
118 might we expect in estimates of animal abundance when classical MRDS
119 models are applied to data like ours? Second, is it possible to mitigate bias
120 in estimates of abundance by explicitly modelling the movement and mea-
121 surement error processes? We now attempt to develop such a framework

122 before conducting some small simulation studies to investigate bias of our
 123 proposed procedure relative to classical approaches that ignore movement
 124 and measurement error.

125 **3. Model development.** Distance sampling surveys seek to estimate
 126 the abundance, N , of an animal population in a given survey region. How-
 127 ever, only a fraction p_c of the survey region is covered by transects, so it is
 128 customary to focus on estimation of animal abundance in the “covered re-
 129 gion,” N_c , and then to use design-based procedures to expand this estimate
 130 to the full study area (e.g. $\hat{N} = N_c/p_c$). In the present paper, we focus on
 131 estimation of N_c , with the understanding that such estimates can easily be
 132 expanded to a larger study area if desired (e.g. [Buckland et al., 2004](#)). For
 133 notational ease, we shall use N to represent N_c for the remainder of the
 134 paper.

135 Consider a double-observer MRDS survey where each observer records
 136 binned distances to detected animal groups, independently of the other ob-
 137 server, and a total of n animal groups are encountered by at least one ob-
 138 server (see [Table 1](#) for a complete list of notation). We develop a two-stage
 139 approach for estimating abundance in the surveyed area from such data.
 140 In the first step, a MML framework is used to simultaneously estimate pa-
 141 rameters of detection, movement, and measurement error processes. In the
 142 second, a Horvitz-Thompson-like estimator is used to estimate abundance
 143 conditioned on parameter estimates from step 1.

144 In MRDS surveys with binned distances, observers record animals as oc-
 145 ccurring in one of $n_{\mathcal{S}}$ perpendicular distance bins, $\mathcal{S} = \mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_{n_{\mathcal{S}}}$. De-
 146 tection probability typically decreases with distance from the transect line,
 147 and the maximum distance bin is often set such that animals farther away
 148 can be ignored without greatly affecting precision of abundance estimates.
 149 Movement and measurement error introduce complications: in addition to
 150 such errors among elements of \mathcal{S} , animals can potentially move into or out
 151 of \mathcal{S} , and animals outside of \mathcal{S} can be detected in \mathcal{S} . For these reasons, the
 152 models we develop rely on augmenting \mathcal{S} with additional distance bins to
 153 allow for movement and measurement error ([Fig. 3](#)). Call this augmented
 154 set \mathcal{Z} .

155 Let y_{io} be a binary indicator for whether or not the i th animal group was
 156 detected by observer o . Similarly, let d_{io} denote the distance bin recorded
 157 by observer o for animal group i (note d_{io} is only defined when $y_{io} = 1$). We
 158 assume that distances are only recorded within the truncation range of the
 159 transect, so $y_{io} = 0$ whenever an animal group is perceived as having $d_{io} \notin \mathcal{S}$.
 160 Letting bold upper case symbols denote matrices (e.g. \mathbf{Y} is a $(2 \times n)$ matrix

161 of all detection/nondetections), we seek to define a joint density function
 162 $[\mathbf{Y}, \mathbf{D}|\boldsymbol{\theta}, \mathbf{X}]$, where $\boldsymbol{\theta} = \{\boldsymbol{\beta}, \phi, \boldsymbol{\varphi}\}$ are parameters describing probabilities of
 163 detection, movement, and measurement error, respectively, and \mathbf{X} includes
 164 individual covariates collected for each animal group that can be used to
 165 explain variation in detection probabilities. Note that here and throughout
 166 the paper, we use the bracket notation (e.g. $[\mathbf{Y}|\mathbf{X}]$) to denote the conditional
 167 distribution of \mathbf{Y} given \mathbf{X} ; we treat responses (e.g. \mathbf{Y}, \mathbf{D}) as random variables
 168 under this construction even though it is not explicitly written.

169 3.1. *Likelihood.* To construct an appropriate likelihood for statistical in-
 170 ference, we start with the general framework proposed by [Borchers et al.](#)
 171 (2015) for spatial mark-recapture and distance sampling surveys. Condition-
 172 ing on detection, [Borchers et al.](#) (2015) suggested that the joint distribution
 173 of animal locations and detections could be written as a product of (1) a
 174 joint probability density function (pdf) for the latent locations of animals,
 175 and (2) a joint probability mass function (pmf) for the encounter histories
 176 conditional on location. We expand upon this framework to allow movement
 177 to affect the distribution of animal locations and to incorporate a measure-
 178 ment error mechanism.

179 Letting z_{io} denote the true location (latent distance bin) of animal group
 180 i when it enters the field of view of observer o , we write the joint probability
 181 mass function of observed data as a product of

- 182 1. $[\mathbf{Z}|\boldsymbol{\theta}]$, a bivariate probability mass function for the distribution of true
 183 animal locations, given detection by at least one observer; and
- 184 2. $[\mathbf{Y}, \mathbf{D}|\mathbf{Z}, \boldsymbol{\theta}, \mathbf{X}]$, a model for binary detections and observed distances
 185 given true unobserved locations and individual detection covariates.

186 If we knew the true locations of observed animals, we could simply base
 187 inference on the likelihood $\mathcal{L}(\boldsymbol{\theta}; \mathbf{Z}, \mathbf{D}, \mathbf{Y}, \mathbf{X})$ with corresponding joint density
 188 function

$$[\mathbf{Z}, \mathbf{D}, \mathbf{Y}|\boldsymbol{\theta}, \mathbf{X}] = [\mathbf{Y}, \mathbf{D}|\mathbf{Z}, \boldsymbol{\theta}, \mathbf{X}][\mathbf{Z}|\boldsymbol{\theta}].$$

189 However, we do not know the true animal locations so instead integrate
 190 (sum) over an augmented set of distance bins \mathcal{Z} that could plausibly have
 191 resulted in a detection (see *Distribution of animal locations* for more dis-
 192 cussion of bin augmentation). As such, we write the marginal likelihood of
 193 detection, movement, and measurement error parameters as $\mathcal{L}(\boldsymbol{\theta}; \mathbf{D}, \mathbf{Y}, \mathbf{X})$,

194 which corresponds to the joint density

$$(3.1) \quad [\mathbf{Y}, \mathbf{D} | \boldsymbol{\theta}, \mathbf{X}] = \prod_i \left(\sum_{z_{i1} \in \mathcal{Z}} \sum_{z_{i2} \in \mathcal{Z}} [y_{i1}, y_{i2}, d_{i1}, d_{i2} | z_{i1}, z_{i2}, \boldsymbol{\theta}, \mathbf{x}_i] [z_{i1}, z_{i2} | \boldsymbol{\theta}] \right).$$

195 Recall that z_{io} is the true distance bin associated with animal group i when
 196 it passes observer o . Similarly, y_{io} gives detection/nondetection, and d_{io} gives
 197 observed distance values (which are missing whenever $y_{io} = 0$), and \mathbf{x}_i is a
 198 vector of covariates for individual i . We now describe each of the likelihood
 199 components in further detail. We shall make an attempt to redefine various
 200 quantities when needed; we also provide parameter definitions in Table 1.

201 3.1.1. *Distribution of animal group locations.* The first component of
 202 the joint density (Eqn. 3.1) is the joint mass function for the locations of
 203 group i , $[z_{i1}, z_{i2} | \boldsymbol{\theta}]$, given detection by at least one observer. We write this
 204 distribution as a function of (i) an initial state distribution, $[z_{i1}]$; (ii) a
 205 movement kernel, $[z_{i2} | z_{i1}, \boldsymbol{\phi}]$; and (iii) detection probability by at least one
 206 observer, $p_i^*(z_{i1}, z_{i2})$. Specifically, we set

$$(3.2) \quad [z_{i1}, z_{i2} | \boldsymbol{\theta}] \propto [z_{i1}] [z_{i2} | z_{i1}, \boldsymbol{\phi}] p_i^*(z_{i1}, z_{i2}),$$

207 where the ‘ \propto ’ sign indicates normalization such that $[z_{i1}, z_{i2} | \boldsymbol{\theta}]$ sums to 1.0.
 208 We explicitly include the thinning probability $p_i^*(z_{i1}, z_{i2})$ in this component
 209 since observations in the data set are conditional on detection, and thus the
 210 position of observed animals are more likely to be close to the transect line
 211 than far away from it.

212 We make the common distance sampling assumption (cf. [Buckland et al.,](#)
 213 [2001](#)) that perpendicular distances of animals groups from the transect line
 214 are uniformly distributed when entering the field of view of the first observer.
 215 This seems reasonable in applications where the first observer has a field
 216 of view facing forward, and when the survey platform moves fast relative
 217 to the speed of focal taxa (as in our waterfowl example; see Discussion for
 218 further consideration of this assumption). Letting π_j denote the proportional
 219 diameter of distance bin j (i.e. $\pi_j = a_j / \sum_k a_k$ where a_j is the diameter of
 220 of distance bin j), the (continuous) uniform distance assumption translates
 221 into the following model for the latent distance bin of animal group i when
 222 encountered by the first observer:

$$z_{i1} \sim \text{Categorical}(\pi_1, \pi_2, \dots, \pi_{n_{\mathcal{Z}}}),$$

223 where it is understood that ‘‘Categorical’’ denotes a multinomial distribution
 224 with index 1, and $n_{\mathcal{Z}}$ is the number of latent distance bins.

225 Next, the bivariate movement pmf $[z_{i2}|z_{i1}, \phi]$ describes the location of
 226 animal group i when it enters the field of view of observer 2 as a function of
 227 the location when it was in the field of view of observer 1. We model this as
 228 another categorical distribution:

$$(3.3) \quad [z_{i2}|z_{i1}, \phi] = \text{Categorical}(\psi(z_{i1}, 1), \psi(z_{i1}, 2), \dots, \psi(z_{i1}, n_Z)).$$

229 For applications in this paper, we parameterize the movement transition
 230 probabilities ψ using asymmetric kernels k (e.g. Fig 4). Using an asymmetric
 231 kernel can allow movement rates to vary based on the direction of animal
 232 movement. In particular, responsive movement away from the transect line
 233 may be more likely than movement towards the transect line. In particular,
 234 we set

$$(3.4) \quad \psi(z_{i1}, z_{i2}) \propto k(z_{i1}, z_{i2}|\phi), \text{ where}$$

$$(3.5) \quad k(z_{i1}, z_{i2}|\phi) = \begin{cases} f(\delta_{i2}|\mu = \delta_{i1}, \sigma = \phi_1) & z_{i2} < z_{i1}, m_i = 1 \\ f(\delta_{i2}|\mu = \delta_{i1}, \sigma = \phi_2) & z_{i2} \geq z_{i1}, m_i = 1 \\ 1.0 & z_{i2} = z_{i1}, m_i = 0 \\ 0.0 & z_{i2} \neq z_{i1}, m_i = 0. \end{cases}$$

238 Here, $f()$ gives a probability density function; in our examples, we consider
 239 Laplace (double exponential) and Gaussian distributions as choices for $f()$.
 240 Note that δ_{io} gives the perpendicular distance from the transect line to the
 241 midpoint of distance bin z_{io} , and that ϕ_1 and ϕ_2 are unknown scale param-
 242 eters of the movement kernels. Also note that we assume that stationary
 243 animals (i.e. with $m_i = 0$) do not change distance bins.

244 Finally, the thinning probability $p_i^*(z_{i1}, z_{i2})$ is the probability of being
 245 detected by at least one observer for an animal that is in distance bin z_{i1} at
 246 time 1 and z_{i2} at time 2. It is a function of both detection probability and
 247 measurement error parameters:

$$p_i^*(z_{i1}, z_{i2}) = 1 - [1 - p_{i1}(z_{i1})\omega(z_{i1}, \mathcal{S})][1 - p_{i2}(z_{i2})\omega(z_{i2}, \mathcal{S})],$$

248 where $p_{io}(z_{io})$ is the probability of observer o detecting group i when it is at
 249 distance z_{io} , and $\omega(z_{io}, \mathcal{S})$ is the probability that the observer perceives the
 250 group to be within the truncation range (\mathcal{S}) of the transect. This expres-
 251 sion is slightly different than typically encountered in MRDS models, as one
 252 must account for two ways of getting a 0 in a capture history: an observer
 253 can either miss the animal group or detect the group but determine it to
 254 be outside the truncation range of the transect (i.e. $\notin \mathcal{S}$). To account for

255 the latter possibility, we make use of the measurement error probabilities
 256 $\omega(z, d)$, which can be parameterized in terms of a measurement error ker-
 257 nel similarly to ψ (see Eqs. 3.4-3.5). Given a true distance bin z , we then
 258 compute $\omega(z, \mathcal{S}) = \sum_{d \in \mathcal{S}} \omega(z, d)$. In applications in the paper, we consider
 259 use of symmetric measurement error kernels (Gaussian or Laplace) with a
 260 single scale parameter, φ .

261 In order to impart meaningful variation in detection probability, it is
 262 useful to express $p_{io}(z_{io})$ in a regression framework on a logit-linear scale,
 263 such that

$$(3.6) \quad \text{logit}(\mathbf{p}) = \mathbf{X}\boldsymbol{\beta}.$$

264 where \mathbf{X} is a design matrix and $\boldsymbol{\beta}$ is a vector of regression parameters. Note
 265 that we shall often include z_{io} (a latent quantity) within \mathbf{X} ; thus the design
 266 matrix \mathbf{X} will not be fixed as is customary in regression applications. For
 267 example, suppose we desire a model where detection probability is a function
 268 of distance, squared distance, observer, and group size (g_i). We could then
 269 formulate Eq. 3.6 such that

$$(3.7) \quad \begin{pmatrix} \text{logit}(p_{11}) \\ \text{logit}(p_{12}) \\ \text{logit}(p_{21}) \\ \text{logit}(p_{22}) \\ \vdots \\ \text{logit}(p_{n1}) \\ \text{logit}(p_{n2}) \end{pmatrix} = \begin{bmatrix} 1 & z_{11} & z_{11}^2 & 0 & g_1 \\ 1 & z_{12} & z_{12}^2 & 1 & g_1 \\ 1 & z_{21} & z_{21}^2 & 0 & g_2 \\ 1 & z_{22} & z_{22}^2 & 1 & g_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & z_{n1} & z_{n1}^2 & 0 & g_n \\ 1 & z_{n2} & z_{n2}^2 & 1 & g_n \end{bmatrix} \times \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix}.$$

270 Note that in the case of irregularly shaped bins, we would likely want to
 271 replace z_{io} in Eq. 3.7 with distance bin midpoints.

272 **3.1.2. Likelihood of observed detections.** The next component of the joint
 273 density function is $[y_{i1}, y_{i2}, d_{i1}, d_{i2} | z_{i1}, z_{i2}, \boldsymbol{\theta}, \mathbf{x}_i]$, the probability of realizing
 274 different random variables for detection (y_{i1} and y_{i2}) and associated dis-
 275 tance bin values (d_{i1} and d_{i2}) for animal group i conditional on true lo-
 276 cation. Conditional on detection by at least one observer, there are three
 277 possible types of encounter histories: $h_i = '11'$ (encountered by both ob-
 278 servers), $h_i = '10'$ (encountered by the first observer and not the second),
 279 and $h_i = '01'$ (encountered by the second observer but not the first). For
 280 '11' histories, there are $n_{\mathcal{S}}^2$ combinations of possible recorded distance bins;
 281 for '10' histories, there are $n_{\mathcal{S}}$ distance bins possible for observer 1; for '01'
 282 histories, there are $n_{\mathcal{S}}$ distance bins possible for observer 2. Thus, we can

view $[y_{i1}, y_{i2}, d_{i1}, d_{i2} | z_{i1}, z_{i2}, \boldsymbol{\theta}, \mathbf{x}_i]$ as a multinomial distribution with index 1 and $n_{\mathcal{S}}^2 + 2n_{\mathcal{S}}$ possible outcomes. The likelihood contribution, L_i , for a particular animal group i can thus be written as $L_i = (p_i^*(z_{i1}, z_{i2}))^{-1} Pr(h_i)$, where the probability of observing each type of history is:

$$\begin{aligned} Pr(h_i = '11') &= p_{i1}(z_{i1})\omega(z_{i1}, d_{i1})p_{i2}(z_{i2})\omega(z_{i2}, d_{i2}), \\ Pr(h_i = '10') &= p_{i1}(z_{i1})\omega(z_{i1}, d_{i1}) [p_{i2}(z_{i2})(1 - \omega(z_{i2}, \mathcal{S})) + (1 - p_{i2}(z_{i2}))], \text{ and} \\ Pr(h_i = '01') &= p_{i2}(z_{i2})\omega(z_{i2}, d_{i2}) [p_{i1}(z_{i1})(1 - \omega(z_{i1}, \mathcal{S})) + (1 - p_{i1}(z_{i1}))]. \end{aligned}$$

For a review of notation, see Table 1.

3.2. Horvitz-Thompson-like abundance estimator. Minimizing the negative log-likelihood associated with Eqn. 3.1 provides marginal maximum likelihood estimates for detection, movement, and measurement error parameters, but does not provide a direct estimate of animal abundance, N . We developed a Horvitz-Thompson-like procedure for N , as is common in distance sampling literature (e.g. Buckland et al., 2004). This is especially useful when coping with detection probabilities that vary as a function of individual detection covariates. For instance, in standard MRDS applications, one might estimate abundance as

$$\hat{N} = \sum_{i=1}^n \frac{g_i}{p_i^*},$$

where n is the number of animals detected, g_i is the number of animals in group i , and p_i^* is the probability of detection by at least one observer. However, direct application of this estimator is clearly inappropriate under movement and measurement error, as it can potentially include animals outside of the surveyed area, or include animals that move into the surveyed area; thus, further development is needed.

We construct a Horvitz-Thompson-like estimator for abundance in the surveyed region \mathcal{S} when animals enter the field of view of observer 1 (i.e. before any movement is assumed to occur) as follows:

$$(3.8) \quad \hat{N} | \hat{\boldsymbol{\theta}} = \sum_i \sum_{z_{i1} \in \mathcal{S}} \sum_{z_{i2} \in \mathcal{Z}} \frac{g_i \times [z_{i1}, z_{i2} | \hat{\boldsymbol{\theta}}]}{p_i^*(z_{i1}, z_{i2})}.$$

This formulation integrates over the latent position of animal groups as they pass observers 1 and 2 (i.e. z_{i1} and z_{i2}) with the restriction that they must be within the truncation range of the transect (i.e. $\in \mathcal{S}$) when they pass observer 1. The quantity $[z_{i1}, z_{i2} | \hat{\boldsymbol{\theta}}]$ gives the discrete probability mass function for the true locations of detected animals evaluated at the MLEs.

311 As suggested by a reviewer, dependence on p_i^* in the denominator of Eq.
 312 3.8 may lead to instability of the estimator if p_i^* is low. In distance sampling
 313 applications, it is thus common to replace p_i^* with an expectation over i ,
 314 $E(p^*)$ (Laake and Borchers, 2004, Eq. 6.11). We leave examination of such
 315 a procedure to future research.

316 To compute variance, we adapt the approximation given independently
 317 by Huggins (1989, 1990) and Alho (1990) and subsequently used by other
 318 authors in distance sampling applications (e.g. Borchers et al., 2006). This
 319 approach uses the law of total variance to write variance as a function of
 320 i) the variance associated with the number of animals encountered, and ii)
 321 variance associated with uncertainty in estimated parameters. Specifically,
 322 our approximation is

$$\begin{aligned} \hat{\text{Var}}(\hat{N}) &= \sum_i \frac{(1 - \tilde{p}_i)g_i}{\tilde{p}^2} + \hat{\mathbf{D}}' \hat{\Sigma} \hat{\mathbf{D}}, \text{ where} \\ \tilde{p}_i &= \sum_{z_{i1} \in \mathcal{S}} \sum_{z_{i2} \in \mathcal{Z}} [z_{i1}, z_{i2} | \hat{\boldsymbol{\theta}}] p_i^*(z_{i1}, z_{i2} | \hat{\boldsymbol{\theta}}). \end{aligned}$$

323 Here, $\hat{\mathbf{D}}$ is a vector of derivatives, $d\hat{N}/d\theta_i |_{\hat{\theta}_i}$, evaluated at the MLEs, and $\hat{\Sigma}$
 324 is the estimated variance-covariance matrix of MRDS parameters. We use
 325 log-based confidence intervals (Burnham et al., 1987, pg. 212) customary in
 326 distance sampling applications (Buckland et al., 2001), and examine preci-
 327 sion and confidence interval coverage of this approximation in all subsequent
 328 simulation analyses.

329 Another possible approach for variance estimation often used in distance
 330 sampling applications is to employ a data-based bootstrap where data are
 331 resampled with replacement, parameters re-estimated, and replicate esti-
 332 mates of \hat{N}_{boot} used to determine a confidence interval (e.g. using the 2.5th
 333 and 97.5th quantiles for a 95% interval). If transects are of equal area, it is
 334 customary to resample data at the *transect* level (rather than the level of
 335 individual observations) to better capture spatial variation associated with
 336 patchy animal distributions. We do not specifically evaluate this procedure;
 337 nevertheless, it may be useful in some applications.

338 **3.3. Extension to incorporate detection heterogeneity.** So far, we have
 339 not attempted to model detection heterogeneity outside of individual co-
 340 variates (e.g. through Eqn. 3.6). However, it is common knowledge that
 341 other factors (e.g. variation in plumage, lighting, topography, background,
 342 etc.) can affect the distinctiveness of different animal groups and impart
 343 additional heterogeneity leading to (often positive) dependence in observer

344 detection and thus negative bias in \hat{N} (Laake and Borchers, 2004; Buckland,
345 Laake and Borchers, 2010; Burt et al., 2014).

346 In traditional MRDS applications (i.e. without movement and measure-
347 ment error), one approach is to correct for this bias by estimating observer
348 dependence parameters, typically by including an additional probability den-
349 sity function for observed distances within a joint likelihood (cf. Buckland,
350 Laake and Borchers, 2010). However, inclusion of such a pdf in our like-
351 lihood appears problematic, as movement alters interpretation of distance
352 distributions (Burt et al., 2014). For instance, movement can induce pat-
353 terns in observed distance distributions that appear similar to those caused
354 by individual heterogeneity. Alternatively, MacKenzie and Clement (2016)
355 suggested that observer dependence could also be included by modeling *con-*
356 *ditional* detection probabilities; that is, including detection by one observer
357 as a covariate for detection of the other. For instance, detection probabili-
358 ties could potentially be written as a logit-linear function of an autocovariate
359 $\xi_{io} = y_{i,3-o}$. We adapt this latter idea as a way to accommodate detection
360 heterogeneity in data subject to movement and measurement error.

361 The major complication with using a detection autocovariate as a predic-
362 tor in our case is that we are no longer able to say that an animal group
363 with $y_{io} = 0$ was actually undetected by observer o . It could, for instance,
364 have been detected but determined to not be in \mathcal{S} . As such, we view the au-
365 tocovariate ξ_{io} as a latent variable. If $y_{io} = 1$, then $\xi_{i,3-o} = 1$ with certainty;
366 however, if $y_{io} = 0$ we do not know whether $\xi_{i,3-o}$ is 0 or 1.

367 An example may help make the preceding points clearer. For instance,
368 suppose two observers gather distance data in an MRDS survey. After the
369 study, a truncation range of 5 distance bins is determined. For a particular
370 animal group, the first observer records a distance bin value of 5 (the distance
371 bin farthest away from the transect), but the second observer either does
372 not detect the animal or perceives the animal to be outside of the 5 distance
373 bins used in the analysis (e.g. distance bin 6). In either case, a non-detection
374 is entered for the second observer when formatting detection histories. Now,
375 suppose that the true distance bin was 5 for both observers (i.e. $z_{i1} = z_{i2} =$
376 5). When modeling this detection probability, we must therefore account for
377 both possibilities for observer 2 when writing the probability of the encounter
378 history and in conditioning on $\xi_{i,3-o}$.

379 To implement this idea, we rewrite the observation model as

$$[y_{i1}, y_{i2}, d_{i1}, d_{i2} | z_{i1}, z_{i2}, \boldsymbol{\theta}, \mathbf{x}_i] = [p_i^*(z_{i1}, z_{i2})]^{-1} Pr(h_i),$$

380 where $Pr(h_i)$ depends on the type of history observed, such that

$$\begin{aligned}
 Pr(h_i = '11') &= p_{i1}(z_{i1}|\xi_{i1} = 1)\omega(z_{i1}, d_{i1})p_{i2}(z_{i2}|\xi_{i2} = 1)\omega(z_{i2}, d_{i2}), \\
 Pr(h_i = '10') &= p_{i1}(z_{i1}|\xi_{i1} = 1)\omega(z_{i1}, d_{i1})p_{i2}(z_{i2}|\xi_{i2} = 1)(1 - \omega(z_{i2}, \mathcal{S})) + \\
 &\quad p_{i1}(z_{i1}|\xi_{i1} = 0)\omega(z_{i1}, d_{i1})(1 - p_{i2}(z_{i2}|\xi_{i2} = 1)), \text{ and} \\
 Pr(h_i = '01') &= p_{i2}(z_{i2}|\xi_{i2} = 1)\omega(z_{i2}, d_{i2})p_{i1}(z_{i1}|\xi_{i1} = 1)(1 - \omega(z_{i1}, \mathcal{S})) + \\
 &\quad p_{i2}(z_{i2}|\xi_{i2} = 0)\omega(z_{i2}, d_{i2})(1 - p_{i1}(z_{i1}|\xi_{i1} = 1)).
 \end{aligned}$$

381 Note that $p_{io}(z_{io}|\xi_{io})$ gives detection probability for observer o for animal
 382 group i , conditional on the animal group being located in distance bin z_{io}
 383 and whether or not it is detected by the other observer. Similarly, recall that
 384 $\omega(z_{io}, d_{io})$ gives the probability that animal group i that is truly in distance
 385 bin z_{io} will be assigned to distance bin d_{io} given it is detected by observer
 386 o .

387 The probability of being detected by at least one observer can be recal-
 388 culated in a similar fashion. However, it is no longer possible to succinctly
 389 write $p_i^*(z_{i1}, z_{i2})$ as the complement of the probability of being undetected.
 390 Instead, we sum the probability of obtaining each encounter history type (h_i
 391 = '11', '01', or '10') (subject to uncertainty about ξ_{io}):

$$\begin{aligned}
 p_i^*(z_{i1}, z_{i2}) &= p_{i1}(z_{i1}|\xi_{i1} = 1)\omega(z_{i1}, \mathcal{S})p_{i2}(z_{i2}|\xi_{i2} = 1)\omega(z_{i2}, \mathcal{S}) + \\
 &\quad p_{i1}(z_{i1}|\xi_{i1} = 0)\omega(z_{i1}, \mathcal{S})(1 - p_{i2}(z_{i2}|\xi_{i2} = 1)) + \\
 &\quad p_{i1}(z_{i1}|\xi_{i1} = 1)\omega(z_{i1}, \mathcal{S})p_{i2}(z_{i2}|\xi_{i2} = 1)(1 - \omega(z_{i2}, \mathcal{S})) + \\
 &\quad p_{i2}(z_{i2}|\xi_{i2} = 0)\omega(z_{i2}, \mathcal{S})(1 - p_{i1}(z_{i1}|\xi_{i1} = 1)) + \\
 &\quad p_{i2}(z_{i2}|\xi_{i2} = 1)\omega(z_{i2}, \mathcal{S})p_{i1}(z_{i1}|\xi_{i1} = 1)(1 - \omega(z_{i1}, \mathcal{S})).
 \end{aligned}$$

392 Following these adjustments, we use the ‘‘symmetric’’ parameterization
 393 (MacKenzie and Clement, 2016) of observer dependence to include ξ_{io} in
 394 the logit-linear model for p_{io} . For instance, point independence (Laake and
 395 Borchers, 2004; Buckland, Laake and Borchers, 2010), where observers are
 396 assumed to detect animal groups independently near the transect line but to
 397 have increasing dependence with distance, can be implemented by including
 398 an interaction between distance and ξ_{io} with no main effect for ξ_{io} . For ex-
 399 ample, in a model including linear and quadratic effects of distance bin (z_{io})
 400 on detection probability, a full independence (‘fi’) model might be written
 401 as

$$\text{logit}(p_{io}) = \beta_0 + \beta_1 z_{io} + \beta_2 z_{io}^2,$$

402 where a point independence (‘pi’) model could be written as

$$\text{logit}(p_{io}) = \beta_0 + \beta_1 z_{io} + \beta_2 z_{io}^2 + \beta_3 z_{io} \xi_{io}.$$

403 Alternatively, limiting dependence (‘li’ [Buckland, Laake and Borchers, 2010](#)),
 404 where there is a base level of dependence on or near the transect line, can
 405 be implemented by including a main effect for ξ_{io} in addition to the interac-
 406 tion ([MacKenzie and Clement, 2016](#)). This could be accomplished with the
 407 formulation

$$\text{logit}(p_{io}) = \beta_0 + \beta_1 z_{io} + \beta_2 z_{io}^2 + \beta_3 \xi_{io} + \beta_4 z_{io} \xi_{io}.$$

408 **3.4. Goodness-of-fit.** Goodness-of-fit is often summarized with χ^2 tests
 409 when distance data are binned ([Burnham et al., 2004](#)). However, this de-
 410 pends on having adequate sample sizes and homogeneous probabilities of
 411 detection within classes of animals. This latter requirement is problematic
 412 when detection probability is written in terms of individual covariates. In-
 413 stead, we developed a simulation-based goodness-of-fit procedure similar in
 414 spirit to posterior predictive checks used in Bayesian analysis (e.g. [Gelman](#)
 415 [et al., 2014](#)). Our procedure consists of

- 416 1. Sampling $\boldsymbol{\theta}_k \sim \text{Multivariate Normal}(\hat{\boldsymbol{\theta}}, \hat{\mathbf{H}}^{-1})$, where $\hat{\boldsymbol{\theta}}$ are maximum
 417 likelihood estimates (MLEs), and $\hat{\mathbf{H}}$ is a matrix of second derivatives
 418 of the likelihood evaluated at the MLEs.
- 419 2. Simulating new data $(\mathbf{d}_k, \mathbf{y}_k)$ from $[\mathbf{d}_k, \mathbf{y}_k | \mathbf{X}, \boldsymbol{\theta}_k]$.
- 420 3. Calculating a discrepancy measure $T(\mathbf{y}, \mathbf{d}, \boldsymbol{\theta})$ to compare the observed
 421 data to data simulated under the model.

422 For instance, we might compute the proportion of observations that occur
 423 in each distance bin when subset by various explanatory variables for our
 424 observed data and compare these to the distribution of proportions that we
 425 obtain by simulating data from our model when all assumptions are met.
 426 For some specific examples, see section 4.

427 **3.5. Computing.** We conducted MML inference in the R programming
 428 environment ([R Development Core Team, 2016](#)). We have collated all code
 429 and data needed to recreate our analyses into an R package, `MRDSmove`. The
 430 package is currently available at [https://github.com/pconn/MRDSmove/](https://github.com/pconn/MRDSmove/releases)
 431 [releases](#) and will be archived in a publicly accessible repository upon
 432 manuscript acceptance.

433 **4. Analysis of waterfowl data.** We fitted 8 MML models to our wa-
 434 terfowl data, varying by (1) movement and measurement kernel type (Gaus-
 435 sian vs. Laplace), (2) observer dependence type (none or point indepen-
 436 dence), and (3) whether or not moving individuals had a different distance
 437 function than individuals that were not moving (Table 2). We did not at-
 438 tempt to fit models with limiting independence, owing to poor simulation

439 performance (see *Simulation Study 2*, below). We calculated marginal AIC
440 to select among these models.

441 All models included the following predictors within the logit-linear model
442 for detection probability: group size, moving/not moving, observer (front
443 vs. back), distance, squared distance, and an interaction between the dis-
444 tance effects and the observer effects. The latter interaction was included
445 because the view of the first distance bin was partially obstructed for ob-
446 server 2 whose distance distribution appeared to peak farther away from the
447 helicopter (see [Alisauskas and Conn, 2017](#)). We applied the same detection,
448 movement, and measurement error parameters to all species as distance dis-
449 tributions appeared similar for all species and previous analysis suggested
450 similar detection profiles ([Alisauskas and Conn, 2017](#)).

451 AIC strongly favored models with Laplace movement and measurement
452 error kernels (Fig. 4) over Gaussian kernels, although the impact of the func-
453 tional form of the kernel on resultant abundance estimates was quite small
454 (Table 2). The highest ranked model had an interaction between distance
455 and moving/not moving, suggesting different detection function shapes for
456 moving vs. stationary animals. However, pairwise model comparisons with
457 and without such an effect had similar AIC scores, so this effect was likely
458 small. Point independence ('pi') was favored over full independence ('fi')
459 models, suggesting some level of detection heterogeneity that was not cap-
460 tured via gathered covariates. As expected, 'pi' models resulted in higher
461 abundance estimates and higher CVs than 'fi' models. In our case, estimates
462 were an average of 13% higher for 'pi' models than 'fi' models (Table 2) and
463 CV increased from 3% to 7%. Plots of movement and measurement error
464 kernels (Fig. 4) for the highest ranked model resembled raw data histograms
465 (Fig. 2). Using the highest-ranked AIC model, densities of waterfowl in the
466 surveyed area ranged from 0.4 individuals/km² for rock ptarmigan to 7.5
467 individuals/km² (Fig. 5).

468 To examine fit of our model to the data, we compared the properties of
469 our MRDS dataset to 1000 data sets simulated from the highest ranked AIC
470 model. In general, data sets simulated under our model had similar propor-
471 tions of animals observed in the five distance bin classes as we observed in
472 the field (Fig. 6). A notable exception was a tendency to overpredict the
473 proportion of moving animals in distance bin 3. We are unsure why there
474 may have been a dip in detections in the third distance bin, but have resisted
475 the urge to consider more highly parameterized structures since a smooth
476 decrease in the number of animals encountered as a function of distance is
477 often expected a priori ([Buckland et al., 2001](#)), and it would be difficult to
478 fit this particular "dip" in our distance data without making the detection

479 model multimodal. Our model did a reasonable job in replicating the pro-
 480 portions of animals with each detection history type observed in the field.
 481 For instance, the number of ‘11’, ‘10’ and ‘01’ histories compiled for moving
 482 animals was 289, 261, and 179, respectively; these compared to 95% simula-
 483 tion intervals of (257,307), (227,276), and (173,219). For stationary animals,
 484 we observed 64 ‘11’, 92 ‘10’ and 79 ‘01’ histories compared to simulation
 485 intervals of (53,80), (74,103), and (68,95).

486 **5. Simulation studies.** We conducted two simulation studies to inves-
 487 tigate bias, precision, and confidence interval coverage of our MML models
 488 and compared these to other MRDS analyses that do not account for move-
 489 ment and measurement error. The first simulation study assumed indepen-
 490 dence between observer detections (i.e. no residual detection heterogeneity).
 491 The second experiment focused on performance of different approaches to
 492 estimation when heterogeneous detection probabilities were simulated using
 493 random effects.

494 *5.1. Simulation study I: Basic model performance.* Our first simulation
 495 study was designed to investigate estimator performance over different move-
 496 ment and measurement error rates, and only considering variation imparted
 497 by measurable covariates. For this study, we simulated three different Gaus-
 498 sian movement kernel (Eqn. 3.5) scenarios, corresponding to (i) no movement
 499 ($\phi_1 = \phi_2 = 0$), (ii) symmetric movement ($\phi_1 = \phi_2 = 0.7$), and (iii) asymmet-
 500 ric movement with much higher rates of movement away from the transect
 501 line than towards the transect line ($\phi_1 = 0.5, \phi_2 = 1.5$). We considered two
 502 levels of measurement error for each movement scenario: no measurement
 503 error, or minor measurement error ($\varphi = 0.5$). The latter value of measure-
 504 ment error was chosen to approximate the level of error we observed in our
 505 waterfowl data.

506 In each of 500 simulations for the 6 movement and measurement error
 507 scenarios, we conducted the following steps:

- 508 1. For each of $i \in 1, 2, \dots, 1000$ animals, we simulated an initial, latent
 509 position z_{i1} in 8 equally sized distance bins using a uniform distribu-
 510 tion.
- 511 2. After generating $m_i \sim \text{Bernoulli}(0.75)$ (so that approximately 75% of
 512 animals were moving), we simulated z_{i2} using Eqn. 3.3. For animals
 513 with $m_i = 0$, we simply set $z_{i2} = z_{i1}$.
- 514 3. We simulated y_{io} and d_{io} using detection and measurement error mod-
 515 els, where the first five distance bins were subject to observation (i.e.

516 $\mathcal{S} = \{\mathcal{Z}_1, \mathcal{Z}_2, \dots, \mathcal{Z}_5\}$. Detection probabilities were configured as

$$\text{logit}(p_{io}) = \beta_0 + \beta_1 m_i + \beta_2 z_{io} + \beta_3 z_{io}^2,$$

517 where $\beta_0 = 1$, $\beta_1 = 0.5$, $\beta_2 = 0.07$, and $\beta_3 = -0.09$.

- 518 4. We fit a sequence of three models to each such data set. These included
 519 (i) the movement and measurement error model proposed in this pa-
 520 per (configured with 8 latent distance bins), as well as two Huggins-
 521 Alho models (HA; Huggins, 1989; Alho, 1990). We fit the HA models
 522 using program MARK (White and Burnham, 1999) via an RMark
 523 (Laake, 2013) interface. The HA models suppose independent detec-
 524 tion of observers and do not account for movement or measurement er-
 525 ror; abundance estimates are generated with a Horvitz-Thompson-like
 526 procedure. The two HA models differed in how distance mismatches
 527 were handled: in the first (HA1), distance was set to d_{i1} whenever
 528 $d_{i1} \neq d_{i2}$ (i.e. using the distance value for observer 1 whenever there is
 529 a mismatch); in the second (HA2), conflicting distance measurements
 530 were averaged. For all three estimation procedures, we used the same
 531 structure when estimating p_{io} as used to generate the data. For sim-
 532 ulations where data were generated with movement or measurement
 533 errors equal to 0.0, we fixed the corresponding parameter in the esti-
 534 mation model to zero to prevent numerical errors.
- 535 5. For each model and data set, we tabulated bias, coefficient of variation
 536 (CV), 95% confidence interval coverage, and root mean square error
 537 (RMSE) of abundance estimators.

538 Note that in initial simulation work, we also fit movement and measurement
 539 error models with 10 latent distance bins, finding that results were almost
 540 identical to those with 8 latent distance bins (parameter estimates were often
 541 within 0.0001 of each other).

542 In general, bias from our new method was close to zero, while positive bias
 543 from the HA models could be substantial when movement and/or measure-
 544 ment error occurred (up to 10%; Table 3). Precision and mean squared error
 545 were always better for the MML models than the HA models, with confi-
 546 dence interval coverage closer to nominal. Coverage was slightly high for the
 547 MML models (0.97-0.98 for a 95% interval), and could be poor (e.g. 75%) for
 548 HA models when sufficient movement or measurement error occurred. Inter-
 549 estingly, HA1 estimates tended to have slightly better properties (lower bias,
 550 better coverage, lower RMSE) than HA2 estimates, suggesting that taking
 551 distance values from observer 1 may be a better strategy than averaging
 552 distance values to resolve discrepancies if one cannot model movement and
 553 measurement error directly.

554 When comparing results, note that the MML likelihood incorporates data
 555 on the distribution of observed distances (via Eq. 3.2) whereas HA models do
 556 not; as such the estimators are not equivalent even in absence of movement
 557 and measurement error.

558 *5.2. Simulation study II: Heterogeneous detection.* In our second simu-
 559 lation scenario, we examined performance of our proposed approach when
 560 MRDS data are simulated with highly heterogeneous detection probabili-
 561 ties. The main structure of our simulations was largely similar to the pre-
 562 ceding section. We considered two different movement and measurement
 563 error scenarios corresponding to (i) no movement or measurement error (i.e.
 564 $\phi_1 = \phi_2 = \varphi = 0$) and (ii) movement away from the survey line ($\phi_1 = 0$,
 565 $\phi_2 = 1.5$) with moderate measurement error ($\varphi = 0.5$). For each of these
 566 scenarios, we considered two different expected sample sizes in the sampled
 567 area: $E(N) = 200$ and $E(N) = 1000$. In each combination of simulation
 568 replicates, we conducted 500 simulations via following steps:

- 569 1. For each of $i \in 1, 2, \dots, E(N)$ animals, we simulated an initial, latent
 570 position z_{i1} in 8 equally sized distance bins using a uniform distribu-
 571 tion.
- 572 2. We generated m_i (a binary indicator for whether individual i is moving
 573 or not) and z_{i2} (latent position of the animal when it passes observer
 574 2) as in Simulation Study 1.
- 575 3. We simulated d_{io} and y_{io} (distance and detection, respectively) as in
 576 simulation study 1, once again using 5 observable distance bins. How-
 577 ever, we used a half-normal model for detection probability,

$$p_{io} = p_{io}^0 \frac{f(z_{io} | \mu = 1, \sigma_{io})}{f(1 | \mu = 1, \sigma_{io})},$$

578 where p_{io}^0 gives detection probability in the first distance bin, and the
 579 half normal model describes how detection probability declines in bins
 580 that are farther away. These models were further parameterized as

$$\begin{aligned} \text{logit}(p_{io}^0) &= \beta_0 + \beta_1 m_i, \text{ and} \\ \log(\sigma_{io}) &= \alpha_0 + \alpha_1 m_i + \epsilon_i, \end{aligned}$$

581 where $\beta_0 = \alpha_0 = 1$, $\beta_1 = 0.5$, $\alpha_1 = 0.2$, and $\epsilon_i \sim \text{Uniform}(-0.7, 0.7)$.
 582 The half-normal model seemed a reasonable way to implement point
 583 independence (Laake and Borchers, 2004) using random effects (Fig.
 584 7).

- 585 4. We fit five models to each such data set. These included the same three
586 models from Simulation Study 1, as well as two marginal likelihood
587 model that attempted to estimate observer dependence parameters in
588 addition to movement and measurement error. In particular, we fit-
589 ted models using point independence (pi) and limiting independence
590 models (li) as described in section 3.3.
- 591
- 592 5. For each model and data set that resulted in a positive definite Hessian
593 matrix, we tabulated bias, coefficient of variation (CV), 95% confidence
594 interval coverage, and root mean square error (RMSE).

595 Simulations suggested that the MML model with point independence did
596 a reasonable job at estimating abundance under all scenarios (Table 3) even
597 though the estimation model differed from the data generating model (poly-
598 nomial vs. half normal detection model; observer dependence effect vs. ran-
599 dom effects) . In particular, bias was low (-0.03 to 0.03) and 95% confidence
600 interval coverage was close to nominal (0.92 - 0.99) for all scenarios ex-
601 amined. In contrast, bias of models ignoring observer dependence could be
602 considerable (up to -9%) with precision that was too high, leading to con-
603 fidence interval coverage that was too low (as low as 8% in one scenario).
604 Not surprisingly, bias was typically negative when ignoring observer depen-
605 dence. However, there was a mediating effect on bias whenever data were
606 simulated subject to both movement, measurement error, and observer de-
607 pendence. Since movement and measurement error alone induce positive
608 bias, and observer dependence alone produces negative bias, the combina-
609 tion of both processes attenuated bias. For instance, HA models performed
610 better when both sources of bias were present than when only one source of
611 bias was present.

612 Models accounting for observer dependence with a limiting independence
613 ('li') formulation performed poorly. These models only resulted in positive
614 definite Hessian matrices in 39% of simulations with low abundance (95%
615 of simulations with high abundance). Those simulations that did converge
616 produced estimators with considerable negative bias (up to 19%), low pre-
617 cision, and poor confidence interval coverage. These models are known to
618 border on nonidentifiability in applications without measurement and move-
619 ment errors (Buckland, Laake and Borchers, 2010; MacKenzie and Clement,
620 2016); evidently, their performance is even further degraded when one must
621 simultaneously account for movement and measurement error. Given this
622 poor performance, we suggest that investigators only employ point indepen-
623 dence ('pi') formulations to address residual detection heterogeneity when
624 movement and/or measurement error models are employed.

625 **6. Discussion.** In this paper, we developed an approach to account for
 626 movement and measurement error in MRDS analyses when observers inde-
 627 pendently record distances to animals, and when there is a binary covariate
 628 for movement. In simulation studies, our approach exhibited low bias and
 629 RMSE when compared to a procedure that ignores movement and measure-
 630 ment error (the latter resulted in positive biases of up to 8%). Importantly,
 631 we were able to conduct estimation even in the face of residual detection het-
 632 erogeneity using a point independence assumption, which seems like a useful
 633 advance. Indeed, estimation of abundance in our field study was much more
 634 sensitive to different functional forms for observer dependence than it was
 635 to different functional forms for movement or measurement error.

636 Our research was motivated by an applied problem: can one obtain reli-
 637 able estimates of bird abundance using aerial survey detections when animals
 638 responsively move away from aircraft? Our results suggest that the answer
 639 to this question is yes, provided that one employ a design where assump-
 640 tion violations are minimized. This is an important finding for bird surveys
 641 in the Arctic and elsewhere, as population managers and conservationists
 642 need unbiased estimates of abundance to make responsible conservation and
 643 management decisions (e.g. for regulation of waterfowl harvests).

644 Several avenues of future research are desirable. First, although our focus
 645 here was on errors in distances, other errors may occur (e.g. errors in group
 646 size determinations, individual covariates, species, etc.). Errors in species
 647 identification can be particularly problematic (e.g. [Conn et al., 2014](#)) and
 648 should ultimately be addressed in multi-species surveys. Second, we have
 649 assumed additive measurement error in the present development; in some
 650 situations, multiplicative measurement error (whereby animals farther away
 651 are subject to greater measurement error; [Borchers et al., 2010](#)) may make
 652 more sense.

653 In this paper we conditioned on binary variables m_i for whether a de-
 654 tected group was moving or not. This approach let us estimate movement
 655 separately from measurement error by making the assumption that animals
 656 with $m_i = 0$ are only subject to measurement error. In other situations and
 657 study taxa (e.g. many marine mammals), all animals may be moving in some
 658 fashion, and thus there may be insufficient data to separate these processes.
 659 In these circumstances, auxiliary data (e.g. animals with known location to
 660 estimate measurement error; cf. [Borchers et al., 2010](#)) will likely be needed
 661 to implement our methods.

662 We made several important assumptions in our modeling efforts that
 663 should be considered by practitioners before using our methods. First, we
 664 assumed that animals were uniformly distributed when detected by the first

665 observer. That is, responsive movement happens *after* detection by the first
666 observer. This assumption seemed reasonable in our application, but may
667 not be a good assumption in other studies. Second, we assumed that the
668 true distance bin was fixed when in the field of view of a given observer. As
669 one reviewer noted, this may not always be the case; when animals move
670 between distance bins, the ultimate probability of detection depends on the
671 entire track of an animal while in the field of view. In such a case, abun-
672 dance estimators may be subject to some bias, the magnitude of which would
673 likely require additional continuous (or approximately continuous) time sim-
674 ulations. Recent research suggests that the magnitude of such bias will likely
675 be worse when the speed of the survey platform is slow relative to the speed
676 of the animal (Glennie, Buckland and Thomas, 2015).

677 One exciting avenue for future research would be to expand our type
678 of modelling approach to allow movement within spatial capture-recapture
679 (SCR; e.g. Borchers and Efford, 2008; Royle et al., 2013) models. The gen-
680 eralized likelihood structure of MRDS and SCR is actually very similar
681 (Borchers et al., 2015; Borchers and Marques, 2017), so incorporating move-
682 ment could likely be accomplished using the same construct in the paper
683 (i.e. by viewing an animals' locations as unobserved latent variables and in-
684 tegrating over all possible sequences of locations). The challenge would likely
685 be a numerical one, as space would need to be increased from one to two
686 dimensions and over a finer mesh, and the temporal dimension would need
687 to increase from two observers to a finite number of sampling occasions. One
688 approach to high dimensional integration would be to adopt a Bayesian per-
689 spective within a data augmentation framework (Royle, Dorazio and Link,
690 2007; Conn, Laake and Johnson, 2012).

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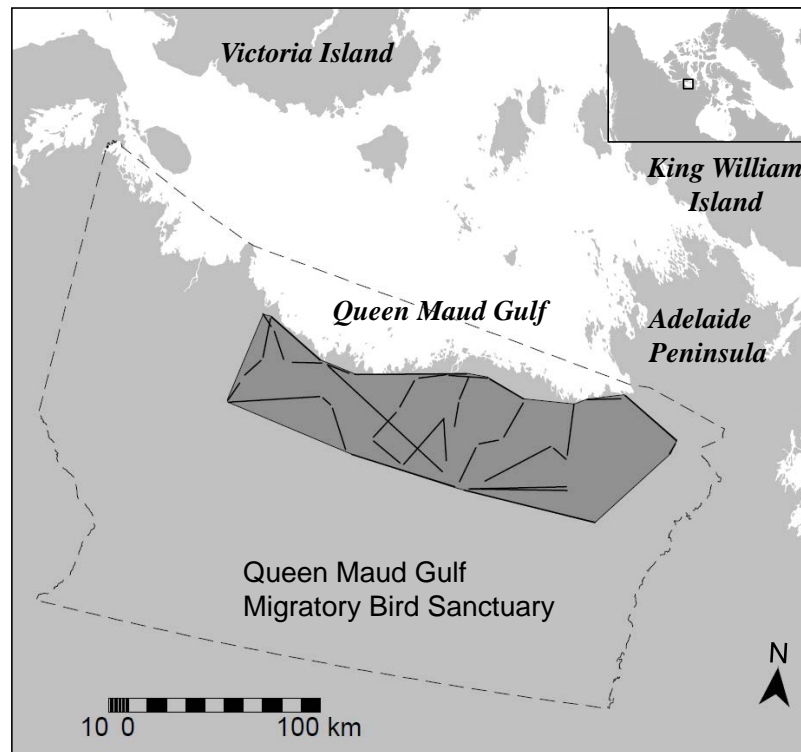


FIG 1. Aerial survey transects (solid lines) and 13,461 km² minimum convex polygon (dark grey) for estimating density and detection probability of 7 bird species in a portion of the Queen Maud Gulf Migratory Bird Sanctuary, Nunavut, Canada.

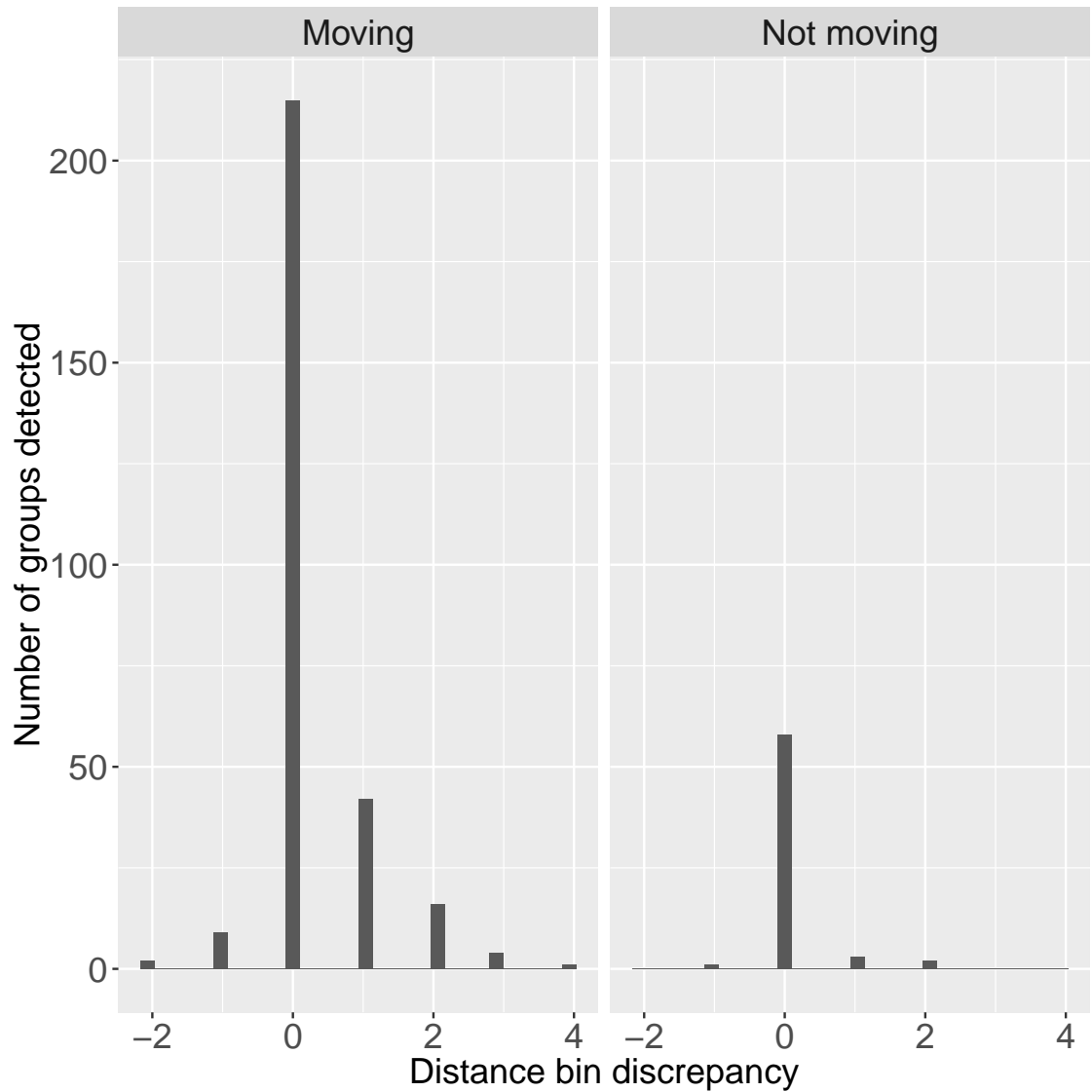


FIG 2. *Distribution of observed distance bin discrepancies for bird groups detected by both observers in helicopter surveys. Negative values imply movement (or measurement error) towards the helicopter, while positive values imply movement away from the helicopter. For moving birds, the distance bin observed by the rear observer tended to be farther away than the bin observed by the front observer. Since the second observer always detected birds later than the front observer, this suggests responsive movement away from the aircraft. For stationary birds, a nonzero distance bin discrepancy represents measurement error.*

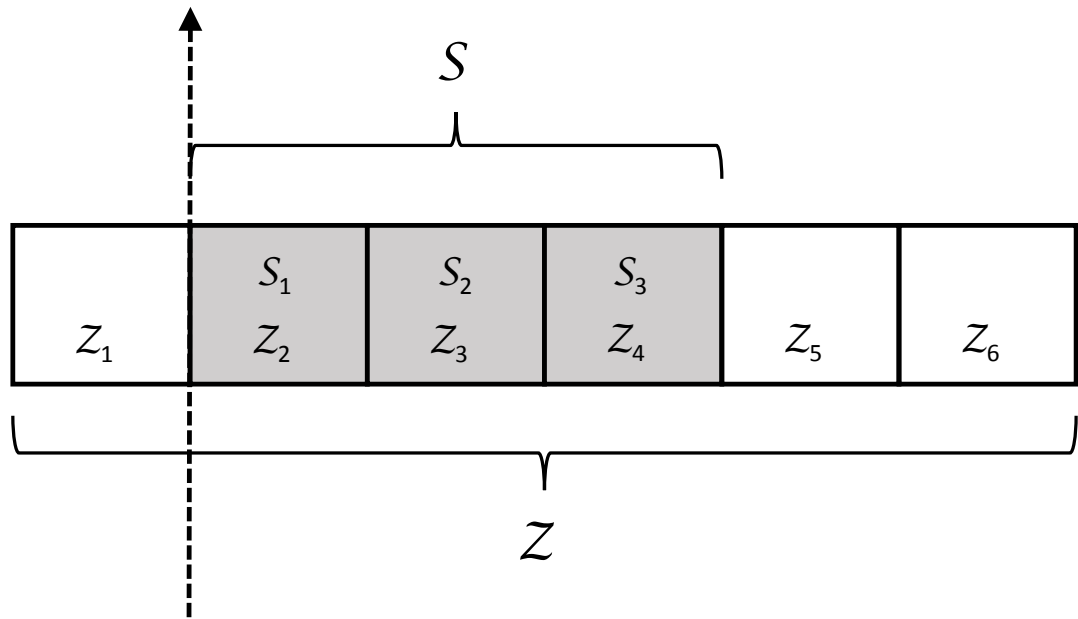


FIG 3. A depiction of observed (S) and latent (Z) distance bins that could potentially be used in analysis of a hypothetical mark-recapture distance sampling (MRDS) survey. In this example, only animals encountered in one of the three shaded distance bins to the right of the transect line (dashed line) are recorded; however, the state space is augmented with an additional three bins to account for possible animal movement and measurement error. In practice, the number of augmented distance bins that are needed will be a function of the magnitude of the movement and measurement error processes.

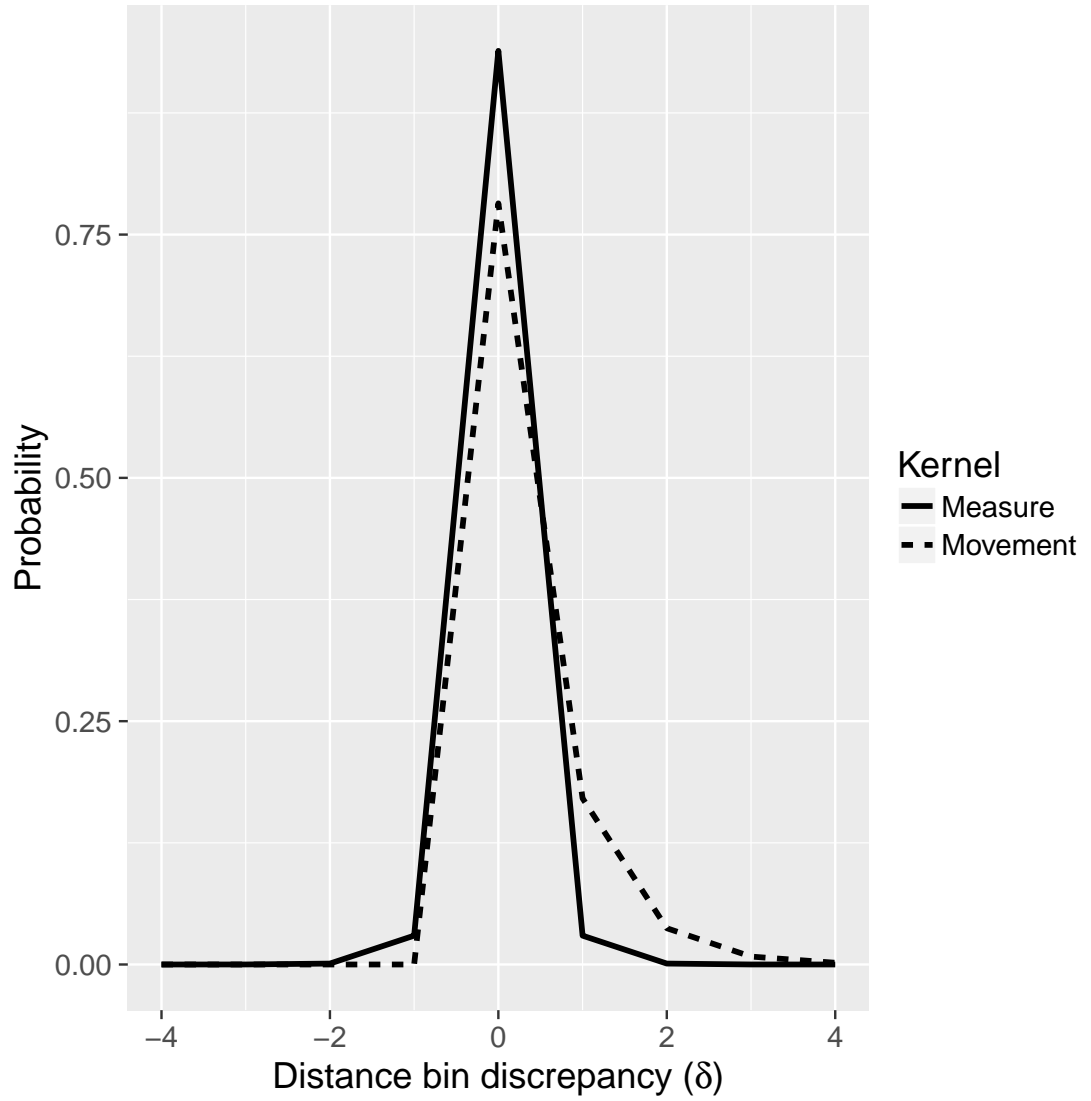


FIG 4. *Estimated movement and measurement error kernels for waterfowl mark-recapture distance sampling (MRDS) data from the highest ranked maximum marginal likelihood model. Measurement error used a (discretized) symmetric Laplace kernel, while movement had an asymmetric Laplace kernel.*

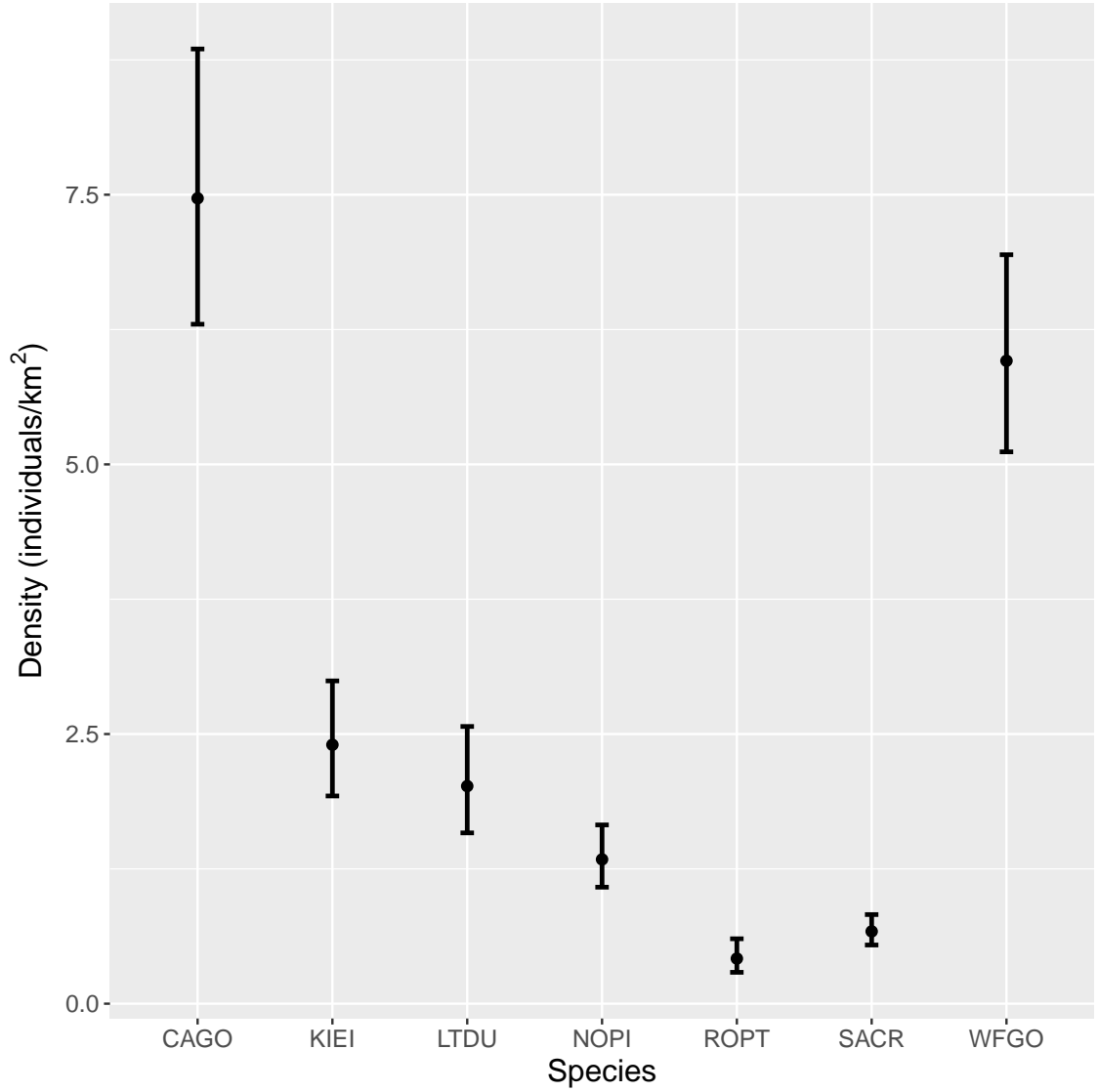


FIG 5. Estimates of bird densities and 95% log-based confidence intervals for the surveyed region in the northern Canada from the highest ranked AIC model. Species included Canada goose (CAGO), king eider (KIED), long-tailed duck (LTDU), northern pintail (NOPI), rock ptarmigan (ROPT), sandhill crane (SACR), and white-fronted goose (WFGO).

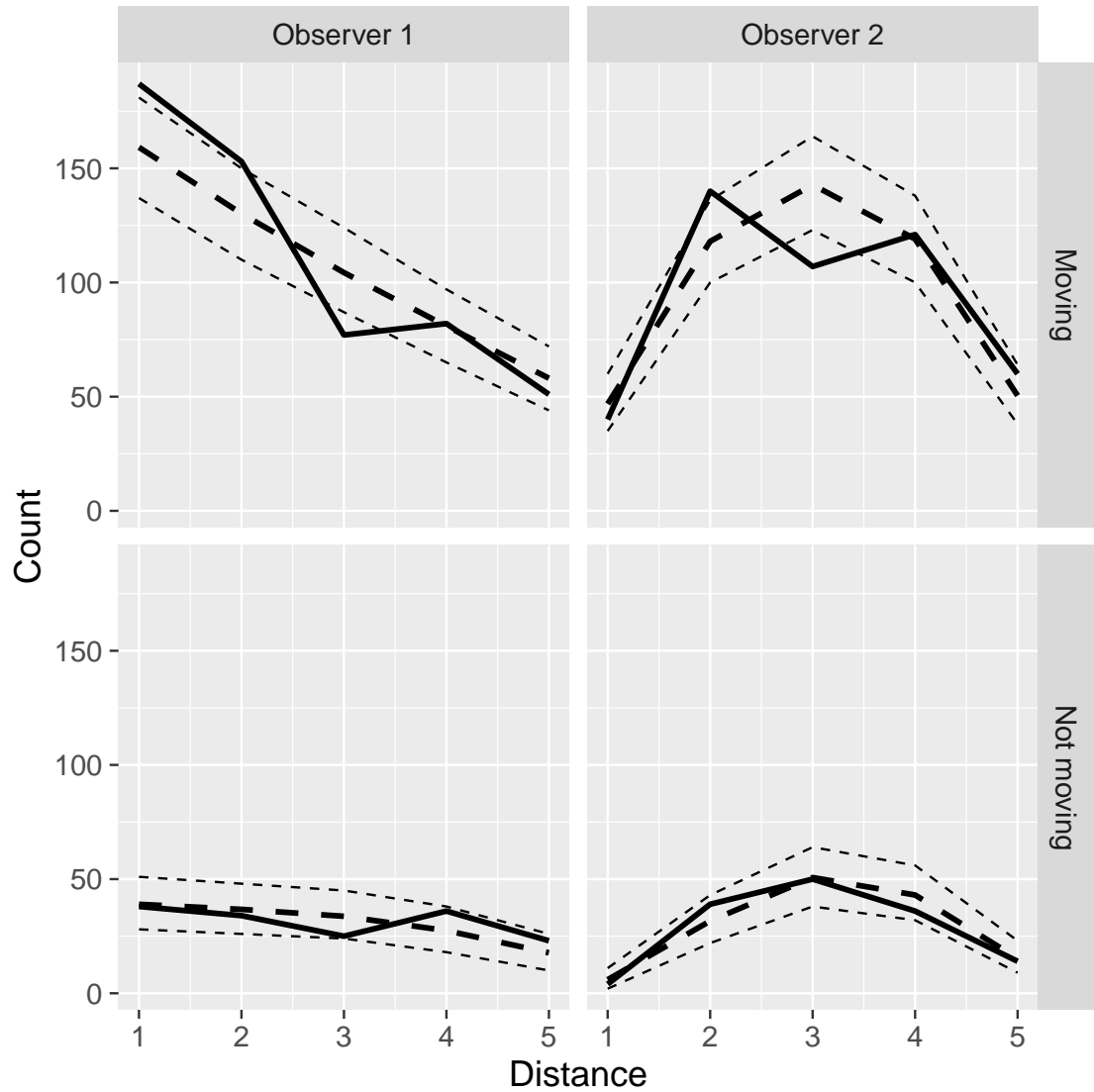


FIG 6. A plot of the number of observed and predicted waterfowl groups by observer and movement status. Observed data are given by the thick solid line, while the thick dashed line represents mean predictions and the thin, dashed lines represent 2.5th and 97.5th quantiles of model-based simulations (including variance due to uncertainty of MML estimates).

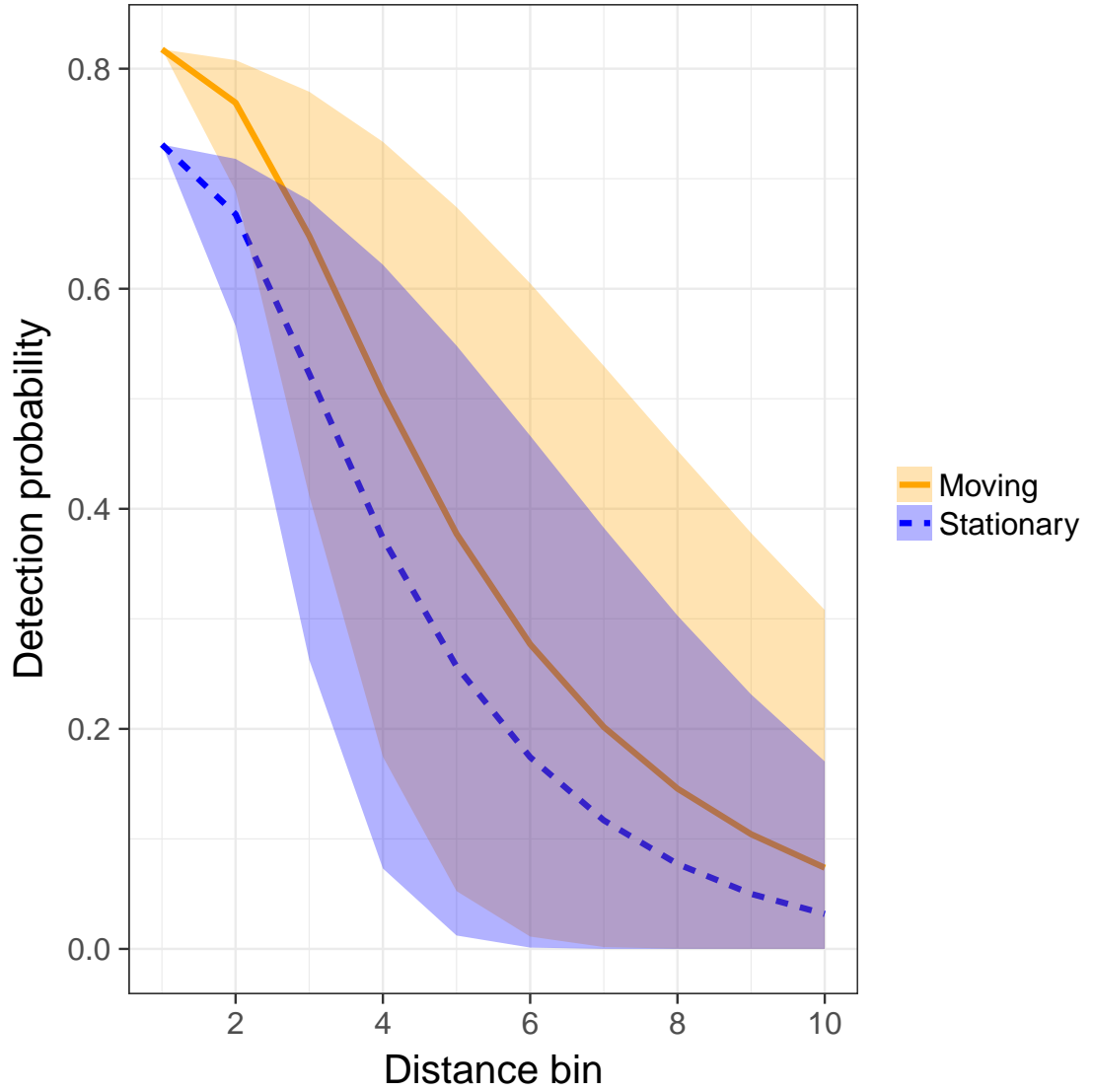


FIG 7. *The simulated range of detection probabilities from Simulation Study 2, where heterogeneity is incorporated via a random effect on the log of the standard deviation associated with a half-normal detection model. Solid and dashed lines represent the expected detection probability for moving and stationary animals, while shaded regions represent 95% intervals.*

TABLE 1

Definitions of fixed and estimated quantities for the double-observer mark-recapture distance sampling (MRDS) model incorporating movement and measurement error.

Quantity	Definition
A. Fixed quantities	
n	Number of animals detected by at least one observer
y_{io}	Binary indicator for whether animal group i was detected by observer o
d_{io}	Distance bin recorded by observer o for animal group i (if recorded)
h_i	Detection history for animal i , obtained by concatenating y_{i1} and y_{i2} (possibilities are '11,' '10,' and '01.')
m_i	A binary indicator for whether animal group i was moving when observed (a single determination is made)
\mathbf{x}_i	A vector of covariates used to explain variation in detection probability for group i
g_i	Number of animals in group i (a single determination is made)
\mathcal{S}	The set of distance bins for which data are recorded, $\mathcal{S} = \mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_{n_{\mathcal{S}}}$. i.e. those bins within the truncation range of the transect.
\mathcal{Z}	The set of latent distance bins used for modelling true animal locations, $\mathcal{Z} = \mathcal{Z}_1, \mathcal{Z}_2, \dots, \mathcal{Z}_{n_{\mathcal{Z}}}$
π_j	Proportion of \mathcal{Z} covered by latent distance bin j
B. Parameters and functions of parameters	
z_{io}	True (latent) distance bin of group i when encountered by observer o
ξ_{io}	An indicator for whether or not observer $3 - o$ detected group i
δ_{io}	Perpendicular distance from the transect line to the midpoint of bin z_{io}
β	A vector of parameters governing logit-linear variation in detection probability
ϕ	Parameters governing animal movement
φ	Parameters governing distance measurement error
θ	The set of detection, movement, and measurement error parameters ($\theta = \{\beta, \phi, \varphi\}$)
$p_{io}(z_{io})$	Probability that observer o detects group i given that the group is truly in distance bin z_{io}
$p_i^*(z_{i1}, z_{i2})$	Probability that at least one observer detects group i given the group is in distance bin z_{i1} at time 1 and z_{i1} at time 2
$\psi(z_{i1}, z_{i2})$	Probability that an animal that is in latent distance bin z_{i1} when it passes observer 1 will be in latent distance bin z_{i2} when it passes observer 2
$\omega(z, d)$	Probability that an animal group in distance bin z is recorded as being in distance bin d
$\omega(z, \mathcal{S})$	Probability that an animal group in distance bin z will have a recorded distance bin falling within \mathcal{S} . Note $\omega(z, \mathcal{S}) = \sum_{d \in \mathcal{S}} \omega(z, d)$.
\mathbf{X}	A design matrix used to impart logit-linear structure on detection probabilities; note this will often include latent distance values, z_{io}
N	True abundance of animals in the surveyed area

TABLE 2

A comparison of models fit to waterfowl data. Models are ranked by AIC; we also provide the number of parameters in each model (k), log likelihood (LogL) at the MLEs, and the total number of focal waterfowl (\hat{N}_c) in the area covered by transects (a sum of species-specific estimates). MML models varied by the functional form of movement and measurement error kernels (Gaussian vs. Laplace), the form of observer dependence (fi: full independence, pi: point independence), as well as whether the detection function included a distance:movement interaction. For reference, the total number of detected birds was 2666.

Model	Δ AIC	k	LogL	$\hat{N}(\hat{SE})$
MML.Laplace.pi.move	0.0	14	-2725.4	3844 (299)
MML.Laplace.pi	3.2	12	-2729.0	3591 (221)
MML.Laplace.fi	6.1	11	-2731.5	3265 (93)
MML.Laplace.fi.move	6.7	13	-2729.8	3288 (96)
MML.Gaussian.pi.move	55.5	14	-2753.2	3827 (310)
MML.Gaussian.pi	58.3	12	-2756.6	3575 (223)
MML.Gaussian.fi	62.9	11	-2760.0	3272 (95)
MML.Gaussian.fi.move	63.6	13	-2758.2	3294 (98)

TABLE 3

Median proportion relative bias (*RelBias*), coefficient of variation (*CV*), 95% confidence interval coverage (*Cover*), and root mean squared error (*RMSE*) for the two simulation studies. For the first simulation scenario, “Configuration” gives values for movement (σ_1 and σ_2) and measurement error (φ) parameters, e.g. (0,0,0), respectively; in simulation study 2, it indicates these parameters as well as expected population size in the surveyed area $N = 200$ or $N = 1000$. Three estimation models (*Model*) were fitted to each data set in simulation study 1: the maximum marginal likelihood (*MML*) model accounting for movement and measurement error, and two Huggins-Alho models which do not account for movement, measurement error, or observer dependence (*HA1* and *HA2*; described in the text). For simulation scenario 2, we fitted *MML* models that embodied full independence (*MML.fi*), point independence (*MML.pi*), and limiting independence (*MML.li*) in addition to the *HA* models.

Configuration	Model	RelBias	CV	Cover	RMSE
A. Simulation study 1					
(0,0,0)	MML	0.01	0.04	0.98	231
(0,0,0)	HA1	0.01	0.04	0.95	553
(0,0,0)	HA2	0.01	0.04	0.95	554
(0.7,0.7,0)	MML	0.01	0.03	0.97	282
(0.7,0.7,0)	HA1	0.03	0.05	0.86	952
(0.7,0.7,0)	HA2	0.05	0.05	0.82	1418
(0.5,1.5,0)	MML	-0.02	0.04	0.91	395
(0.5,1.5,0)	HA1	0.06	0.07	0.80	2887
(0.5,1.5,0)	HA2	0.06	0.07	0.83	2691
(0,0,0.5)	MML	0.00	0.04	0.97	246
(0,0,0.5)	HA1	0.01	0.05	0.93	813
(0,0,0.5)	HA2	0.03	0.05	0.90	1182
(0.7,0.7,0.5)	MML	0.00	0.04	0.98	353
(0.7,0.7,0.5)	HA1	0.03	0.05	0.83	1120
(0.7,0.7,0.5)	HA2	0.06	0.06	0.75	2482
(0.5,1.5,0.5)	MML	0.00	0.04	0.97	379
(0.5,1.5,0.5)	HA1	0.06	0.06	0.83	2607
(0.5,1.5,0.5)	HA2	0.08	0.07	0.81	3831
B. Simulation study 2					
(0,0,0), $N = 200$	MML.pi	0.03	0.11	0.99	444
(0,0,0), $N = 200$	MML.li	-0.11	0.35	0.89	1578
(0,0,0), $N = 200$	MML.fi	-0.04	0.06	0.83	195
(0,0,0), $N = 200$	HA1	-0.07	0.06	0.83	300
(0,0,0), $N = 200$	HA2	-0.07	0.06	0.83	300
(0,1.5,0.5), $N = 200$	MML.pi	-0.02	0.13	0.94	672
(0,1.5,0.5), $N = 200$	MML.li	-0.10	0.39	0.87	2734
(0,1.5,0.5), $N = 200$	MML.fi	-0.08	0.07	0.72	390
(0,1.5,0.5), $N = 200$	HA1	0.00	0.14	0.95	7996
(0,1.5,0.5), $N = 200$	HA2	-0.02	0.12	0.94	2201
(0,0,0), $N = 1000$	MML.pi	0.02	0.05	0.98	2098
(0,0,0), $N = 1000$	MML.li	-0.11	0.23	0.78	36197
(0,0,0), $N = 1000$	MML.fi	-0.06	0.03	0.35	3769
(0,0,0), $N = 1000$	HA1	-0.08	0.02	0.19	6550
(0,0,0), $N = 1000$	HA2	-0.08	0.02	0.19	6550
(0,1.5,0.5), $N = 1000$	MML.pi	-0.03	0.06	0.92	3468
(0,1.5,0.5), $N = 1000$	MML.li	-0.17	0.22	0.69	47853
(0,1.5,0.5), $N = 1000$	MML.fi	-0.09	0.03	0.08	8917
(0,1.5,0.5), $N = 1000$	HA1	-0.02	0.05	0.93	2216
(0,1.5,0.5), $N = 1000$	HA2	-0.04	0.04	0.86	3067