

DISCUSSION OF: ESTIMATING THE HISTORICAL AND FUTURE PROBABILITIES OF LARGE TERRORIST EVENTS

BY GEORGE MOHLER

Santa Clara University

I congratulate [Clauset and Woodard \(2013\)](#) on a very interesting article. The authors analyze a global terrorism dataset with the aim of quantifying the probability of historical and future catastrophic terrorism events. Using power law, stretched exponential, and log-normal tail probability models for the severity of events (# deaths), the authors make a convincing argument that a 9/11 sized event – 2749 deaths, the largest in the dataset – is not an outlier amongst the catalog of terrorist events between 1968 and 2007. This study builds upon earlier work by [Clauset, Young and Gleditsch \(2007\)](#) that I also recommend for those interested in the statistical modeling of terrorism.

While there seems to be consensus amongst the models that 9/11 is not an outlier, there seems to be less one can say with certainty on how likely a 9/11 sized event is. In Table 1 the authors present historical probability estimates (90% confidence interval) for a 9/11 sized event occurring between 1968 and 2007 for each of the three models: power law [.18, .67], stretched exponential [.12,.27], and log-normal [.06,.17]. Likelihood ratio tests fail to pick one model over the others, thus each estimate must be given consideration. Turning to Table 2, where the authors forecast the probability of a 9/11 sized event in 2012-2021, forecasted probabilities range from 0.04 to .94 depending on the model and the frequency of events over the time window. Here the uncertainty has less to do with the model specification and more to do with uncertainty in the frequency of events over the next decade. Terrorist events do not follow a stationary Poisson process and the intensity can fluctuate greatly over a several year period of time.

The authors remark in their discussion that relaxing the stationarity assumptions and incorporating spatial and exogenous variables may help tighten the range of forecasted probabilities. I would add here that some progress has been made in this regard, in particular on modeling regional terrorist event time series as non-stationary point processes (see [Porter and White \(2012\)](#); [Lewis et al. \(2012\)](#); [Zammit-Mangion et al. \(2012\)](#); [Mohler](#)

(2013); Raghavan, Galstyan and Tartakovsky (2012)). Terrorism event processes are history dependent and intensities exhibit correlations at timescales of weeks or months due to self-excitation (see Porter and White (2012); Lewis et al. (2012)) and exogenous effects (see Raghavan, Galstyan and Tartakovsky (2012); Mohler (2013)). Most work has considered the severity distribution to be independent of the intensity and further research is needed to evaluate this assumption. Whether the occurrence of a severe event inhibits the intensity due to military/government intervention, excites the intensity as terrorists attempt to replicate successes, or neither will influence the probability of catastrophic events over short and intermediate timescales and should be reflected in models of terrorist activity.

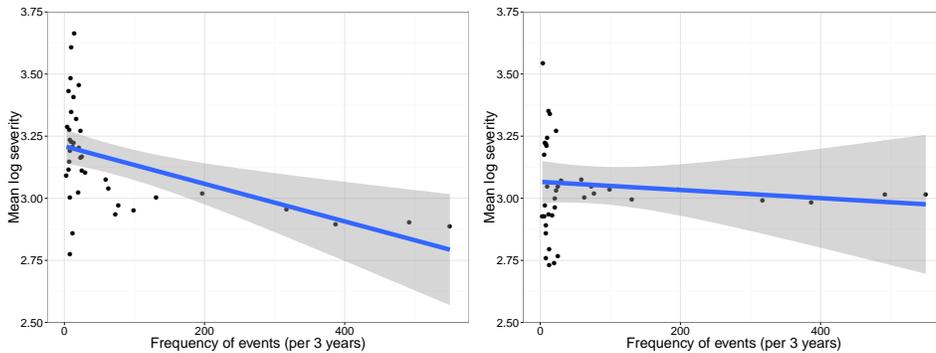


FIG 1. *Left: Mean log severity vs. frequency of events (3 year moving window over 1968 to 2008) for events in RAND-MIPT dataset with severity greater than equal to 10 (slope p -value .002). Right: Mean log severity vs. frequency of events (3 year moving window) for simulated events from a process with frequency distribution independent from severity power law distribution with $\alpha = 2.4$, $x_{min} = 10$ (slope p -value .558).*

One quick way to check for possible dependency between the severity distribution and the rate of events is to calculate the mean log severity of events over a moving window as is done by Clauset, Young and Gleditsch (2007) and compare this distribution against the frequency of events over the same moving window. For example, for the years 1970-2008 we calculate for each year y the mean log severity of events in the time window $[y - 2, y]$ with severity greater than or equal to 10. We plot this distribution (Figure 1) against the frequency of events in each of the moving windows and observe a statistically significant negative correlation ($p = .002$) between mean log severity and frequency. One possible explanation is that periods of higher event frequency coincide with heightened security measures that reduce the severity of events, though determining causality is difficult as both

the severity and intensity depend on exogenous unobservable variables.

One question that came to mind upon reading the paper is: if 9/11 is not an outlier, what is? Under the power law model that the authors consider, the MLE scaling parameter $\alpha = 2.4$ corresponds to a probability of .3 that at least one 9/11 sized event occurs in a sample of size 994 ($x_{min}=10$). A “rare” event would thus correspond to a much lower probability and in Figure 2 (right) we plot the severity of the sample max as a function of the probability of occurrence and the scaling parameter. For $\alpha = 2.4$, the 95th

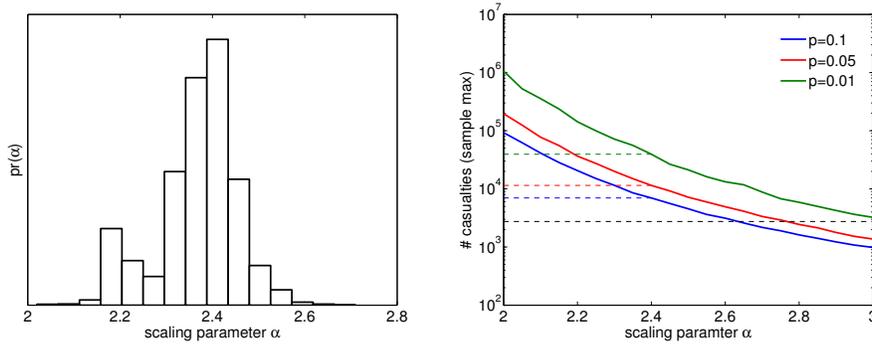


FIG 2. *Left: Bootstrap probability distribution of estimated scaling parameter α , estimated jointly with x_{min} from entire RAND-MIPT dataset of 13,274 events. Right: 90th, 95th and 99th percentiles of the sample max (sample size 994, $x_{min} \geq 10$) corresponding to varying scaling parameter α .*

percentile of the sample max is approximately 12,000 and the 99th percentile is approximately 37,000, an order of magnitude larger than 9/11 (dashed line in Figure 2). The authors note uncertainty in the estimate for α , in particular 15% of bootstrapped estimates cluster around $\alpha = 2.2$ (see Figure 2 left). A decrease in the scaling parameter corresponds to an increase in the severity threshold separating rare from plausible. The 95th and 99th percentiles for $\alpha = 2.2$ correspond to severities of 37,000 and 130,000 respectively. Clauset, Young and Gleditsch (2007) observe that the scaling parameter also varies with time (see Figure 4 in their paper), with the estimated α falling below 2 for short periods of time. Given the sensitivity of event severity to changes in the scaling parameter, short and intermediate term forecasts of catastrophic event probabilities need to take into account the temporal dynamics of α .

The probability of catastrophic terrorist events also depends on the type of weapon used. Clauset and Woodard (2013) find that the estimated historical probability of a 9/11 sized event is greatest for explosives ($p = .37$), fire ($p = .14$), and firearms ($p = .12$). Given that 9/11 is categorized as “other”

and involved a high degree of planning and coordination, it may not be the case that other types of terrorism can be modeled alongside it as iid random variables. In this case it becomes more likely that 9/11 is an outlier, given that the confidence interval the authors provide for the probability of a catastrophic event of type other is $[0, .24]$ and the mean is $p = .06$. Ideally model development should be done in collaboration with domain experts, who can provide insight into whether weapon-dependent severity probabilities are realistic. For example, it may not be plausible for an attack of type firearm to produce a 9/11 (or greater) sized event and a model that places high probability of a catastrophic event for these weapon types should be rejected.

References.

- CLAUSET, A. and WOODARD, R. (2013). Estimating the historical and future probabilities of large terrorist events. *The Annals of Applied Statistics*.
- CLAUSET, A., YOUNG, M. and GLEDITSCH, K. S. (2007). On the frequency of severe terrorist events. *Journal of Conflict Resolution* **51** 58–87.
- LEWIS, E., MOHLER, G., BRANTINGHAM, P. J. and BERTOZZI, A. L. (2012). Self-exciting point process models of civilian deaths in Iraq. *Security Journal* **25** 244–264.
- MOHLER, G. (2013). Modeling and estimation of multisource clustering in crime and security data. *The Annals of Applied Statistics*.
- PORTER, M. D. and WHITE, G. (2012). Self-exciting hurdle models for terrorist activity. *The Annals of Applied Statistics* **6** 106–124.
- RAGHAVAN, V., GALSTYAN, A. and TARTAKOVSKY, A. G. (2012). Hidden Markov Models for the Activity Profile of Terrorist Groups. *arXiv preprint arXiv:1207.1497*.
- ZAMMIT-MANGION, A., DEWAR, M., KADIRKAMANATHAN, V. and SANGUINETTI, G. (2012). Point process modelling of the Afghan War Diary. *Proceedings of the National Academy of Sciences* **109** 12414–12419.

SANTA CLARA UNIVERSITY
E-MAIL: gmohler@scu.edu