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A NOTE ON "BAYESIAN NONPARAMETRIC ESTIMATORS DERIVED FROM CONDITIONAL GIBBS STRUCTURES"

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The present note aims at clarifying some possibly confusing notation used in [1], even if its correct interpretation should be clear from context and from the proofs contained therein. In particular, the expression

$$(*) \qquad \mathbb{P}[K_n = k, \boldsymbol{N}_n = (n_1, \ldots, n_{K_n})]$$

displayed in (3) of [1] has been used to indicate the probability of observing a specific realization of a random partition of the integers $[n] = \{1, \ldots, n\}$ into k blocks of sizes $(n_1, \ldots, n_k) \in \Delta_{n,k}$. However, the notation in $(*)$ may be actually misleading since its natural interpretation is as the probability of all partitions of $[n] = \{1, \ldots, n\}$ into k blocks of sizes $(n_1, \ldots, n_k) \in \Delta_{n,k}$. For this reason, (3) in [1] should be rewritten as

$$(3) \qquad \Pi_k^{(n)}(n_1, \ldots, n_k) = \frac{\theta^k}{(\theta)_n} \prod_{j=1}^{k} (n_j - 1)!.$$

where $\Pi_k^{(n)}$ is the *exchangeable partition probability function* notation introduced in Section 2. Hence, if Π_n is a random element taking values in the set of all partitions of $[n]$, one has $\Pi_k^{(n)}(n_1, \ldots, n_k) = \mathbb{P}[\Pi_n = \pi]$ for any partition π of $[n]$ into k blocks $\{A_1, \ldots, A_k\}$ with $|A_i| = n_i$, for $i = 1, \ldots, k$. See also the first displayed equation on page 1522 after (4) in [1]. The probability of all partitions of $[n]$ into k blocks with respective sizes $n_1, \ldots, n_k$ (in exchangeable order) is then equal to $n!\,\Pi_k^{(n)}(n_1, \ldots, n_k)/(k! n_1! \cdots n_k!)$. The same caveat also applies to (28) and (44)–(46) in [1].

An analogous clarification concerns

$$(**) \qquad \mathbb{P}[K_m^{(n)} = k,\, L_m^{(n)} = s,\, \boldsymbol{S}_{L_m^{(n)}} = (s_1, \ldots, s_{K_m^{(n)}}) \,|\, K_n = j]$$

displayed in (9) of [1]. Indeed, $(**)$ has been used to denote the probability of a specific partition of $[s]$ into k blocks with sizes $(s_1, \ldots, s_k)$ in $\Delta_{s,k}$, given

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any partition of $[n]$ into j parts, thus differing from its natural meaning as the probability of all such partitions. Our intepretation of $(**)$ in [1] is clearly consistent with the right hand side of (9), i.e.

$$\frac{V_{n+m,j+k}}{V_{n,j}} \binom{m}{s} (n-j\sigma)_{m-s} \prod_{i=1}^{k} (1-\sigma)_{s_i-1}$$

and with the result displayed in Corollary 1, where equation (10) arises after multiplying the right hand side of (9) by $s!/(k!s_1! \cdots s_k!)$ and, then, marginalizing with respect to all $(s_1, \ldots, s_k)$ in $\Delta_{s,k}$. Similar remarks apply to the notation appearing in the following displayed formulae: (17), (19), equation at the end of page 1528, (21), equation right after Corollary 2 on page 1529, (22), equation at the end of page 1531, (34).

REFERENCES

[1] Lijoi, A., Prünster, I. and Walker, S.G. (2008). Bayesian nonparametric estimators derived from conditional Gibbs structures. *Ann. Appl. Probab.* **18**, 1519–1547.

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