Correction Note: Rejoinder to "A Significance Test for the Lasso"

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In this note we correct a statement made in our rejoinder to the discussions of "A Significance Test for the Lasso", *The Annals of Statistics*, Vol. 42, No. 2, pages 518–531, 2014.

The discussion of Professors Buhlmann, Meier, and van de Geer provides an alternate form for the covariance test statistic. Sticking to the notation in our original paper, the quantity that they consider is

$$T(A, \lambda_{k+1}) = \Big(\|y - X_A \tilde{\beta}_A(\lambda_{k+1})\|_2^2 + \lambda_{k+1} \|\tilde{\beta}_A(\lambda_{k+1})\|_1 \Big) / \sigma^2 - \\ \Big(\|y - X \hat{\beta}(\lambda_{k+1})\|_2^2 + \lambda_{k+1} \|\hat{\beta}(\lambda_{k+1})\|_1 \Big) / \sigma^2.$$

(In Buhlmann et al. the quantities A and λ_{k+1} above are written as $\hat{A}_{k-1}$ and $\hat{\lambda}_{k+1}$.) This quantity is a difference in criterion values at λ_{k+1}, between $\tilde{\beta}_A(\lambda_{k+1})$ and $\hat{\beta}(\lambda_{k+1})$, but it is important to emphasize that these *are not the criteria used to define* $\tilde{\beta}_A(\lambda_{k+1})$ *and* $\hat{\beta}(\lambda_{k+1})$*, respectively.* To be clear, the two sets of lasso coefficients are defined as

$$\tilde{\beta}_A(\lambda_{k+1}) = \underset{\beta_A \in \mathbb{R}^{|A|}}{\operatorname{argmin}} \ \frac{1}{2} \|y - X_A \beta_A\|_2^2 + \lambda_{k+1} \|\beta_A\|_1,$$
$$\hat{\beta}(\lambda_{k+1}) = \underset{\beta \in \mathbb{R}^p}{\operatorname{argmin}} \ \frac{1}{2} \|y - X \beta\|_2^2 + \lambda_{k+1} \|\beta\|_1,$$

where the leading factor of $\frac{1}{2}$ in front of the sum of squares term is critical. In our rejoinder, we missed this subtlety, and incorrectly concluded that the alternate form of the statistic put forth by Buhlmann et al. was wrong. We apologize to the authors for this oversight, and thank Sara van de Geer for her generosity and patience in pointing it out to us!